

Characteristics of Statistical Thinking in Empirical Enquiry

by

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Abstract

There is an increasing emphasis in teaching, to develop students' capacity to think statistically. Thus, my thesis was undertaken to make explicit, and to document, the reasoning and thinking processes used by students and statisticians in applied statistics. It is an investigation into the nature of statistical thinking in the broad problem solving domain from problem formulation to conclusions.

The research is based around four exploratory studies. In the first two studies statistics students were given tasks ranging from textbook-type questions to newspaper articles. The third and fourth studies involved interviewing professional statisticians and undergraduate statistics students about their approach to statistical problem solving in projects they had undertaken.

Data were collected through recorded interviews. A qualitative research approach was used in each of the four exploratory studies and involved an ongoing analysis and interpretation of the data. Some of the qualitative data were analysed using software to aid the extraction of common themes. Other researcher and interviewee corroboration of the findings were used where possible.

From this research I have posited a four-dimensional statistical thinking framework for empirical enquiry. The dimensions are: the investigative cycle; the interrogative cycle; types of thinking; and dispositions. An inherently statistical way of thinking was identified as 'transnumeration' (a coined word). Other specifically statistical ways of thinking, such as taking variation into account, and the synthesising of context and subject knowledge, were found. These corroborated with other literature sources and therefore this thesis elaborates and extends this knowledge base with particular regard to the role of explanation or causation. Dispositions necessary for good statistical thinking are discussed in relation to statistics. An interrogative cycle has been created to explain how the identified generic thinking skills are specifically used in statistical thinking. Other types of thinking identified have been categorised as reasoning with models, strategic thinking and using techniques.

From all these elements a comprehensive grounded theory on the nature of statistical thinking in the broad problem solving domain has been developed from the data and literature. The implications arising from this theory for teaching are discussed, together with possible solutions based on the development of thinking tools.

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