Characteristics of Statistical Thinking in Empirical Enquiry

by

Maxine Jeanette Pfannkuch

A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Statistics, The University of Auckland, 1999
Abstract

There is an increasing emphasis in teaching, to develop students’ capacity to think statistically. Thus, my thesis was undertaken to make explicit, and to document, the reasoning and thinking processes used by students and statisticians in applied statistics. It is an investigation into the nature of statistical thinking in the broad problem solving domain from problem formulation to conclusions.

The research is based around four exploratory studies. In the first two studies statistics students were given tasks ranging from textbook-type questions to newspaper articles. The third and fourth studies involved interviewing professional statisticians and undergraduate statistics students about their approach to statistical problem solving in projects they had undertaken.

Data were collected through recorded interviews. A qualitative research approach was used in each of the four exploratory studies and involved an ongoing analysis and interpretation of the data. Some of the qualitative data were analysed using software to aid the extraction of common themes. Other researcher and interviewee corroboration of the findings were used where possible.

From this research I have posited a four-dimensional statistical thinking framework for empirical enquiry. The dimensions are: the investigative cycle; the interrogative cycle; types of thinking; and dispositions. An inherently statistical way of thinking was identified as ‘transnumeration’ (a coined word). Other specifically statistical ways of thinking, such as taking variation into account, and the synthesising of context and subject knowledge, were found. These corroborated with other literature sources and therefore this thesis elaborates and extends this knowledge base with particular regard to the role of explanation or causation. Dispositions necessary for good statistical thinking are discussed in relation to statistics. An interrogative cycle has been created to explain how the identified generic thinking skills are specifically used in statistical thinking. Other types of thinking identified have been categorised as reasoning with models, strategic thinking and using techniques.

From all these elements a comprehensive grounded theory on the nature of statistical thinking in the broad problem solving domain has been developed from the data and literature. The implications arising from this theory for teaching are discussed, together with possible solutions based on the development of thinking tools.
Acknowledgements

I wish to acknowledge and thank the many people who have supported me during the writing of this thesis:

My supervisor and colleague Chris Wild for his quiet, constant, positive support, for the debating of ideas, for his openness and for his determination to see this thesis to fruition.

My first supervisor and colleague Constance Brown for the rewarding first exploratory study conversations and latterly for the comments on each chapter.

The statisticians who volunteered to be interviewed, thoroughly checked the interpretation of their interviews, and gave positive feedback.

The statistics project student who volunteered to be interviewed and then checked the interpretation of her interview. All the other statistics students who gave up their time voluntarily to help in my research.

My colleagues in the Mathematics Education Unit for their unfailingly support especially Bill Barton and Mike Thomas who undertook extra work in order to help me complete my thesis.

Andy Begg for allowing me access to his considerable collection of statistical education papers in the early stages of my research.

The many people who have encouraged and helped me: Jill Ellis, Kath and John Truran, Mike Shaughnessy, Garry Tee, Cliff Konold, Rolf Biehler, Heinz Steinbring, Andrea Peter, Werner Schmidt.
I dedicate this thesis to my husband,

Kurt Pfannkuch

and to my parents,

Don and Jean Fisher

for their invaluable love and support.
Table of Contents

Abstract ............................................................................................................................. ii
Acknowledgements ........................................................................................................ iii
Dedication .......................................................................................................................... iv
Table of Contents ........................................................................................................... v
List of Figures ................................................................................................................... xi
List of Tables ..................................................................................................................... xiii

Chapter One

Introduction
1.1 Introduction and Background .................................................................................. 1
1.2 The Need for the Study ............................................................................................. 2
1.3 The Research Question .............................................................................................. 3
1.4 Overview of Chapters ............................................................................................... 3

Chapter Two

Historical Overview: The Emergence of Statistical Thinking
2.1 Introduction ............................................................................................................... 5
2.2 Early History from the Renaissance Period ............................................................... 6
2.3 Early Attempts to Reconcile the Mathematical Definition and the Statistical Definition of Probability ................................................................. 8
2.4 The Transition towards Inferential Statistics ............................................................. 9
2.4.1 Breaking with the Gambling Interpretation of Probability .............................. 9
2.4.2 The Statistical Interpretation of Probability .................................................... 9
2.4.3 Recognition of the Power of Statistics to Reveal ..................................... 10
2.5 Breaking the Social Science Barrier ..................................................................... 11
2.5.1 Expanding the Frequentist Interpretation of Statistical Probability .......... 11
2.5.2 Moving from Data to Statistical Model ...................................................... 12
2.5.3 Interpreting the Statistical Model ................................................................. 12
2.6 The Recognition of Variation ............................................................................... 13
2.7 The Re-evaluation of Statistical Thinking .............................................................. 15
2.8 Summary ................................................................................................................... 16
Chapter Three
Literature Review

3.1 Introduction .............................................................. 17
3.2 Mathematical Problem Solving Perspective ...................... 19
  3.2.1 Influences on Mathematical and Statistical Problem Solving ... 19
  3.2.2 Metacognition and Reasoning in Statistics ................... 21
3.3 Psychologists’ Perspective ........................................... 23
  3.3.1 General .............................................................. 23
  3.3.2 Representativeness ................................................. 24
  3.3.3 Availability ........................................................ 27
  3.3.4 The Role of Intuition .............................................. 29
3.4 Thinking in a Data-Based Environment - Educationists’ Perspective ... 31
  3.4.1 General .............................................................. 31
  3.4.2 Student Thinking in a Data-Based Environment .......... 32
  3.4.3 Statistical Literacy ............................................... 34
  3.4.4 Instruction ........................................................ 37
  3.4.5 Misconceptions .................................................. 39
  3.4.6 The Need for Thinking Tools ................................... 41
  3.4.7 Probabilistic and Deterministic Thinking .................... 42
3.5 Thinking in a Data-Based Environment - Statisticians’ Perspective ..... 46
  3.5.1 General .............................................................. 46
  3.5.2 Quality Management Perspective ............................. 48
  3.5.3 Epidemiology Perspective ...................................... 50
  3.5.4 Causation and Variation ....................................... 52
  3.5.5 The Nature of Statistical Thinking ........................... 54
  3.5.6 The Need for Statistical Thinking ............................ 58
3.6 Current Theoretical Models for Stochastic Thinking .............. 59
  3.6.1 Scholz Model ...................................................... 59
  3.6.2 Other Cognitive Models ....................................... 61
  3.6.3 Epistemological Considerations ............................. 64
3.7 Summary .................................................................... 66

Chapter Four
Research Design

4.1 Research Design Assumptions ....................................... 70
4.2 Researcher Biases ...................................................... 71
4.3 Methods for Overcoming Research Design Biases ............... 72
4.4 Ethics ................................................................. 73
4.5 Research Design ...................................................... 73
  4.5.1 Design Decisions at the Beginning of the Research ....... 73
4.5.1.1 Exploratory Study One ........................................ 73
4.5.2 Design Decisions made Throughout the Research .......... 76
4.5.2.1 Exploratory Study Two ........................................ 76
4.5.2.2 Exploratory Study Three: The Statistician’s Perspective .................................................. 78
4.5.2.3 Exploratory Study Four: The Student-as-Investigator Perspective .................................................. 79
4.5.3 Design Decisions made at the End of the Research .......... 80

Chapter Five
The First Exploratory Study: Building up a Picture on Some Characteristics of Statistical Thinking
5.1 Introduction .............................................................. 81
5.2 Phase One: First Interview ............................................ 82
5.3 Phase Two: The One-Day Course .................................... 87
5.4 Phase Three: The Follow-Up Interview ............................ 91
5.5 Conclusion ............................................................... 95

Chapter Six
The Second Exploratory Study: Building up a Bigger Picture on Some Characteristics of Statistical Thinking
6.1 Introduction .............................................................. 97
6.2 How do Students Interpret Some Aspects of Statistical Information? .. 98
6.3 First Interview ........................................................... 99
6.4 Second Interview ....................................................... 112
6.5 Some Conjectures ...................................................... 115
6.6 Conclusion ............................................................... 120
6.7 Summary ................................................................. 121

Chapter 7
The Third Exploratory Study: Themes Gleaned from Professional Statisticians
7.1 Introduction .............................................................. 123
7.1.1 Background ........................................................... 123
7.1.2 The Statisticians ...................................................... 124
7.1.3 Organisation .......................................................... 125
7.2 Environmental Realities ............................................... 127
7.2.1 Defining the Role of the Statistician ............................. 127
7.2.2 Constraints ........................................................... 128
7.2.3 Gaining and Maintaining Client Trust .......................... 129
### 7.2.4 Managing the Client's Expectations ............................................. 131
### 7.2.5 Ethical Problems ........................................................................... 131

### 7.3 Elements of Statistical Thinking ....................................................... 132
#### 7.3.1 Foundations .................................................................................. 132
#### 7.3.2 Dispositions .................................................................................. 137

### 7.4 Problem ............................................................................................ 138
#### 7.4.1 Introduction .................................................................................. 138
#### 7.4.2 Grasping the Dynamics of the System .......................................... 140
#### 7.4.3 Defining the Problem .................................................................... 141
#### 7.4.4 Factors Affecting Perceptions of the Problem ............................. 142

### 7.5 From Plan to Data ............................................................................ 143
#### 7.5.1 Measurement ................................................................................. 143
#### 7.5.2 The Psychology of Data ................................................................. 145
#### 7.5.3 Design ............................................................................................ 150
#### 7.5.4 Anticipation .................................................................................... 151
#### 7.5.5 Data Production ............................................................................. 154
#### 7.5.6 Criticising and Cleaning Data ...................................................... 155

### 7.6 From Analysis to Conclusions ............................................................ 156
#### 7.6.1 Planned and Unplanned Analyses ................................................. 156
#### 7.6.2 Towards Conclusions .................................................................... 161
#### 7.6.3 Communicating Conclusions .......................................................... 163

### 7.7 Discussion .......................................................................................... 165

#### Chapter Eight

**The Fourth Exploratory Study: Towards Characterising Statistical Thinking**

#### 8.1 Introduction ....................................................................................... 168
#### 8.2 Initial Characterisation ...................................................................... 169
#### 8.3 Final Characterisation ........................................................................ 172
##### 8.3.1 Examples of Analysis .................................................................... 173
#### 8.4 A Return to Exploratory Studies One and Two ..................................... 184
##### 8.4.1 Exploratory Study Two Examples of Analysis ............................... 184
##### 8.4.2 Exploratory Study One Examples of Analysis .................................. 188

#### 8.5 Discussion .......................................................................................... 191

#### Chapter Nine

**Statistical Thinking in Empirical Enquiry**

#### 9.1 Introduction ....................................................................................... 193
#### 9.2 A Statistical Thinking Framework for Empirical Enquiry ..................... 196
##### 9.2.1 Dimension One: The Investigative Cycle ....................................... 197
### Appendix One
**First Exploratory Study Interview Tasks and Short Course Outline**
- A1.1 First Interview Tasks .......................................................... 280
- A1.2 Second Interview Tasks ........................................................ 284
- A1.3 Short Course Outline .......................................................... 288

### Appendix Two
**Second Exploratory Study Interview Tasks**
- A2.1 First Interview Tasks .......................................................... 293
- A2.2 Second Interview Tasks ........................................................ 296

### Appendix Three
**Third Exploratory Study Interview Protocol and Analysis**
- A3.1 Statistician Interview Protocol ............................................ 301
- A3.2 Analysis Node Categories .................................................... 303
- A3.3 Analysis of Statisticians’ Interviews .................................... 307
- A3.4 Cross-Analysis ................................................................. 315

### Appendix Four
**Fourth Exploratory Study Interview Protocol and Analysis**
- A4.1 Project Student Interview Protocol ...................................... 317
- A4.2 Analysis of Project Students’ Interviews ............................... 319

### Appendix Five
**Second Exploratory Study Scoring Rubrics and Responses to Questions**
- A5.1 Map Question Rubric ............................................................ 321
- A5.2 Error Rate Question Rubric .................................................. 324
- A5.3 Die Question Rubric ............................................................. 327
- A5.4 Prison Newspaper Questions 1 & 2 Rubric ............................ 328
- A5.5 Prison Newspaper Question 4 Rubric .................................... 332
- A5.6 Fitness Newspaper Question Rubric ........................................ 336
- A5.7 Analysis of Student Responses ............................................. 341
List of Figures

Figure 2.1 Galton’s Two-stage Quincunx ................................................ 15
Figure 3.1 Schematic View of Statistical Determinism ......................... 44
Figure 3.2 Scholz Model of Stochastic Thinking of a Person .................. 60
Figure 3.3 Epistemological Triangle .................................................... 65
Figure 5.1 Map Question ................................................................... 85
Figure 5.2 Test Results Question ......................................................... 86
Figure 5.3 Error Rate Question ........................................................... 92
Figure 6.1 Map Question ...................................................................101
Figure 6.2 Error Rate Question ...........................................................102
Figure 6.3 Prison Newspaper Question .................................................105
Figure 6.4 Fitness Newspaper Question .................................................112
Figure 6.5 Genetic Damage Question ...................................................113
Figure 8.1 Map Question ...................................................................184
Figure 8.2 Error Rate Question ...........................................................186
Figure 8.3 Test Results Question .........................................................190
Figure 9.1 A 4-dimensional Framework for Statistical Thinking in Empirical
Enquiry .................................................................................196
Figure 9.1(a) Dimension 1: The Investigative Cycle ..............................196
Figure 9.1(b) Dimension 2: Types of Thinking ......................................196
Figure 9.1(c) Dimension 3: The Interrogative Cycle ..............................196
Figure 9.1(d) Dimension 4: Dispositions ..............................................196
Figure 9.2 The Setting for Statistical Investigation ...............................198
Figure 9.3 Interplay between Context and Statistics ..............................200
Figure 9.3(a) From Inkling to Plan ......................................................200
Figure 9.3(b) Shuttling between Spheres ..............................................200
Figure 9.4 Learning via Statistics .........................................................202
Figure 9.5 Using any Technique ..........................................................203
Figure 9.6 The Interrogative Process ....................................................206
Figure 9.6(a) Interrogative Cycle ..........................................................206
Figure 9.6(b) Distillation and Encapsulation .........................................206
Figure 9.7 Sources of Variation in Data .................................................209
Figure 9.8 Practical Responses to Variation ..........................................210
Figure 9.9 Responses to Regularities ...................................................211
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 9.10</td>
<td>Drilling Down into the “Plan” Node of PPDAC</td>
<td>222</td>
</tr>
<tr>
<td>Figure 9.11</td>
<td>Drilling Down further into the “Measurement” Node of “Plan”</td>
<td>223</td>
</tr>
<tr>
<td>Figure 10.1</td>
<td>Student Responses to Map Question</td>
<td>233</td>
</tr>
<tr>
<td>Figure 10.2</td>
<td>Student Responses to Error Rate Question</td>
<td>234</td>
</tr>
<tr>
<td>Figure 10.3</td>
<td>Student Responses to Die Question</td>
<td>234</td>
</tr>
<tr>
<td>Figure 10.4</td>
<td>Student Responses to Prison Newspaper Questions 1 &amp; 2</td>
<td>235</td>
</tr>
<tr>
<td>Figure 10.5</td>
<td>Student Responses to Prison Newspaper Question 4</td>
<td>235</td>
</tr>
<tr>
<td>Figure 10.6</td>
<td>Student Responses to Fitness Newspaper Question</td>
<td>236</td>
</tr>
<tr>
<td>Figure 10.7</td>
<td>Criteria Responses for JOY</td>
<td>240</td>
</tr>
<tr>
<td>Figure 10.8</td>
<td>Criteria Responses for NORS</td>
<td>241</td>
</tr>
<tr>
<td>Figure 10.9</td>
<td>Criteria Responses for MORTA</td>
<td>242</td>
</tr>
<tr>
<td>Figure 10.10</td>
<td>Criteria Responses for EAGLE</td>
<td>243</td>
</tr>
<tr>
<td>Figure 10.11</td>
<td>Criteria Responses for ISA</td>
<td>244</td>
</tr>
<tr>
<td>Figure 10.12</td>
<td>Criteria Responses for TEP</td>
<td>246</td>
</tr>
<tr>
<td>Figure 10.13</td>
<td>Student Response Data for Judgement Criteria</td>
<td>248</td>
</tr>
<tr>
<td>Figure 10.14</td>
<td>Student Response Data Grouped into Four Categories</td>
<td>249</td>
</tr>
<tr>
<td>Figure A1.1</td>
<td>Stem and Leaf Plots on Reading Scores</td>
<td>281</td>
</tr>
<tr>
<td>Figure A1.2</td>
<td>Map Question</td>
<td>282</td>
</tr>
<tr>
<td>Figure A1.3</td>
<td>Test Results Question</td>
<td>282</td>
</tr>
<tr>
<td>Figure A1.4</td>
<td>Error Rate Question</td>
<td>285</td>
</tr>
<tr>
<td>Figure A1.5</td>
<td>Snowflake Patterns 1</td>
<td>286</td>
</tr>
<tr>
<td>Figure A1.6</td>
<td>Snowflake Patterns 2</td>
<td>287</td>
</tr>
<tr>
<td>Figure A1.7</td>
<td>Intuitions and Statistical Thinking</td>
<td>288</td>
</tr>
<tr>
<td>Figure A1.8</td>
<td>Using Intuitions</td>
<td>288</td>
</tr>
<tr>
<td>Figure A1.9</td>
<td>Intuitive Models and Thinking</td>
<td>290</td>
</tr>
<tr>
<td>Figure A2.1</td>
<td>Map Question</td>
<td>293</td>
</tr>
<tr>
<td>Figure A2.2</td>
<td>Error Rate Question</td>
<td>293</td>
</tr>
<tr>
<td>Figure A2.3</td>
<td>Prison Newspaper Question</td>
<td>294</td>
</tr>
<tr>
<td>Figure A2.4</td>
<td>Fitness Newspaper Question</td>
<td>295</td>
</tr>
<tr>
<td>Figure A2.5</td>
<td>Air Crash Newspaper Question</td>
<td>298</td>
</tr>
<tr>
<td>Figure A2.6</td>
<td>Genetic Damage Question</td>
<td>299</td>
</tr>
</tbody>
</table>
## List of Tables

<table>
<thead>
<tr>
<th>Table Number</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>First Exploratory Study Subject Data</td>
<td>74</td>
</tr>
<tr>
<td>4.2</td>
<td>Second Exploratory Study Subject Data</td>
<td>76</td>
</tr>
<tr>
<td>4.3</td>
<td>Third Exploratory Study Subject Data</td>
<td>79</td>
</tr>
<tr>
<td>4.4</td>
<td>Fourth Exploratory Study Subject Data</td>
<td>80</td>
</tr>
<tr>
<td>6.1</td>
<td>Investigating Relationships Question</td>
<td>114</td>
</tr>
<tr>
<td>8.1</td>
<td>Four-dimensional Analysis: Beth Excerpt One</td>
<td>174</td>
</tr>
<tr>
<td>8.2</td>
<td>Four-dimensional Analysis: Beth Excerpt Two</td>
<td>175</td>
</tr>
<tr>
<td>8.3</td>
<td>Four-dimensional Analysis: Beth Excerpt Three</td>
<td>177</td>
</tr>
<tr>
<td>8.4</td>
<td>Four-dimensional Analysis: Isa Excerpt One</td>
<td>185</td>
</tr>
<tr>
<td>8.5</td>
<td>Four-dimensional Analysis: Isa Excerpt Two</td>
<td>187</td>
</tr>
<tr>
<td>8.6</td>
<td>Alcohol and Smoking Status for Pregnant Women</td>
<td>188</td>
</tr>
<tr>
<td>8.7</td>
<td>Four-dimensional Analysis: Ebe Excerpt One</td>
<td>189</td>
</tr>
<tr>
<td>8.8</td>
<td>Four-dimensional Analysis: Ebe Excerpt Two</td>
<td>190</td>
</tr>
<tr>
<td>8.9</td>
<td>Four-dimensional Analysis: Ebe Excerpt Three</td>
<td>191</td>
</tr>
<tr>
<td>10.1</td>
<td>Question Data for JOY</td>
<td>239</td>
</tr>
<tr>
<td>10.2</td>
<td>Question Data for NORS</td>
<td>241</td>
</tr>
<tr>
<td>10.3</td>
<td>Question Data for MORTA</td>
<td>242</td>
</tr>
<tr>
<td>10.4</td>
<td>Question Data for EAGLE</td>
<td>243</td>
</tr>
<tr>
<td>10.5</td>
<td>Question Data for ISA</td>
<td>244</td>
</tr>
<tr>
<td>10.6</td>
<td>Question Data for TEP</td>
<td>245</td>
</tr>
<tr>
<td>A1.1</td>
<td>Smoking and Alcohol Data for Pregnant Women</td>
<td>280</td>
</tr>
<tr>
<td>A1.2</td>
<td>Data on Traffic Accident Deaths</td>
<td>284</td>
</tr>
<tr>
<td>A2.1</td>
<td>Birth Data Information</td>
<td>300</td>
</tr>
<tr>
<td>A3.1</td>
<td>Cross-Analysis of Statisticians’ Interviews</td>
<td>315</td>
</tr>
<tr>
<td>A5.1</td>
<td>Student Responses to Map Question</td>
<td>341</td>
</tr>
<tr>
<td>A5.2</td>
<td>Student Responses to Error Rate Question</td>
<td>341</td>
</tr>
<tr>
<td>A5.3</td>
<td>Student Responses to Die Question</td>
<td>341</td>
</tr>
<tr>
<td>A5.4</td>
<td>Student Responses to Prison Newspaper Questions 1 &amp; 2</td>
<td>342</td>
</tr>
<tr>
<td>A5.5</td>
<td>Student Responses to Prison Newspaper Question 4</td>
<td>342</td>
</tr>
<tr>
<td>A5.6</td>
<td>Student Responses to Fitness Newspaper Question</td>
<td>342</td>
</tr>
</tbody>
</table>