# Chapter Eleven Discussion on the Framework

At the most recent meeting of ICOTS, held in June 1998 in Singapore, several papers focused on the related topics of Statistical Reasoning, Statistical Thinking, and Statistical Literacy. There seemed to be an overlap among the topics, yet important distinctions between them, none of which have as yet been addressed. It became apparent that when statistics educators or researchers talk about or assess statistical reasoning, thinking, or literacy, they may all be using different definitions and understandings of these cognitive processes. The similarities and differences among these processes are important to consider when formulating learning goals for students, designing instructional activities, and evaluating learning by using appropriate assessment instruments. In addition, in recent years, we have seen an increasing research emphasis on the socially and culturally situated nature of mathematical (statistical) activity. It suggests the importance of participation in the statistical practices established by the classroom community, in scaffolding the statistical reasoning processes of the individual student. A small, focused conference consisting of researchers interested in these topics appears to be an important next step in clarifying the issues, connecting researchers and their studies, and generating some common definitions, goals, and assessment procedures. (Ben-Zvi and Garfield, personal communication, 1998)

#### **11.1 Introduction**

From the body of literature that I have surveyed there appears to be no comprehensive theoretical framework developed for statistical thinking. Scholz (1987) proposed a model for stochastic thinking while other researchers have used general models such as the SOLO taxonomy model (e.g. Watson et al., 1994) or have adapted mathematical frameworks (e.g. Nitko & Lane, 1992). Thus the proposed framework is a first attempt at constructing an overall view of the many dimensions involved in operationalising the complex activity of statistical thinking in empirical enquiry. The development of the framework culminated from an ongoing interpretive process based on empirical data from four exploratory studies that focussed on the statistics discipline itself. The data were interpreted from the perspectives of a teacher-practitioner, and a statistician. Thus when making comparisons with other models it is important to realise there will be differences

in approach dependent upon the perceptions and field of the researcher. The paradigms of psychologists, sociologists, educational theorists and quality managers will be different as each field will interpret data from its distinctive perspective. In the discussion that follows I will attempt to demonstrate that our statistical thinking framework, which attempts to model practitioner behaviour, is in consonance with the elements proposed, or used by, other researchers.

#### **11.2** Comparison with Other Models

The Scholz (1991) model would appear to be closely related to the interrogative cycle dimension. It mainly describes the way a person interacts with a statistical problem in the analysis and conclusion stages of the investigative cycle. Scholz found students used an intuitive and analytical mode. The intuitive mode is used for easily accessible knowledge. The analytical mode requires the students to do a systematic search of their knowledge base to access higher ordered knowledge. From the student interviews in the second exploratory study we found, similarly, that knowledge was drawn from a reaction to what was said about the problem, rather than from a proactive production of knowledge that could solve, or bring other context and statistical knowledge to the problem. Scholz conjectured that there needed to be some sort of switch to trigger the analytical mode. In our model this teaching strategy idea is found in several places. The switch he refers to could be encouraged by developing the interrogative cycle dimension into a thinking tool, similar to that described in Section 9.4 and Chapter 10. Another teaching strategy idea is found in the epistemological triangle of Steinbring (1991). This model reveals that, in teaching a concept such as variation, a linkage must be built up between the contextual and statistical knowledge base, through exposure to a rich array of problem situations. The framework captures these aspects under 'reasoning with statistical models' and 'integrating context and statistical knowledge'.

Watson et al.'s (1994) work uses the SOLO taxonomy model to describe the type and level of cognitive functioning occurring in students. This perspective provides a general way of assessing students' cognitive development from a unistructural level to a relational level. Using their model Watson et al. found that reasoning in statistics requires multimodal functioning in the ikonic and concrete-symbolic areas and is influenced by the context of the problem. In our model, these facets are brought out in 'reasoning with statistical models' and in the 'integration of context and statistical knowledge'. In our 'interrogative cycle' and 'dispositions' dimensions, the role of 'imagination' is highlighted, which is regarded as an important feature in Watson et al.'s research findings on the use of the ikonic mode in statistics. Shaughnessy (1992) characterised student growth in conceptions of statistics in four levels. The highest level is pragmatic-statistical

which is incorporated in the framework as 'techniques' and 'reasoning with statistical models'. The modelling and quantifying of variation is a distinctive part of the Shaughnessy model albeit with different terminology.

The Nitko and Lane (1992) framework for statistical assessment tasks is in consonance with our framework at a more generalised level, as specific elements of statistical thinking such as 'transnumeration', 'variation', the 'search for causes' and the 'investigative cycle' are not highlighted as assessment areas. Their statistical domains of problem solving, and statistical modelling and argumentation, would be classified under our 'investigative cycle' and 'types of thinking' dimensions. Their cognitive domains of representation, knowledge structure, connections among types of knowledge, active construction of knowledge and situated cognition would be classified under 'types of thinking' and 'interrogative cycle' dimensions. Again the purpose of their framework is different from our framework and therefore is focussed on aspects that are pertinent to assessment.

The framework of Ben-Zvi and Friedlander (1996) describes the different levels of thinking that were achieved by students in the carrying out of statistical investigations. Our framework does not describe levels of thinking rather it focuses on the type of thinking that should be prompted in statistical investigations. When making a comparison, their levels of thinking are an elaboration of elements that we have identified. Their framework can be recognised and described from our model perspective in all dimensions. For example, in our model: Mode 0: Uncritical thinking would be basic engagement in the 'interrogative cycle' with the 'interpret' mode being only operationalised to the 'describe' stage not to 'multiple explanations'; *Mode 1: Meaningful* use of representation would include 'techniques', 'reasoning with statistical models', a poor 'integration of context and statistical knowledge' and a limited notion of 'variation'; Mode 2: Meaningful handling of multiple representations: developing metacognitive abilities would be an example of 'transnumeration', 'criticise' in the 'interrogative cycle', the 'investigative cycle' and the 'integration of context and statistical knowledge'; *Mode 3*: Creative Thinking would be classified under the 'conclusions-communication' stage of the 'investigative cycle' dimension and under 'transnumeration' in the 'types of thinking' dimension. Mention is made in their research that the context of the investigation affects the Modes of thinking employed, which is in agreement with our 'dispositions' dimension.

In comparing our framework with other models we believe that our perspective is different, as we have attempted to model how a statistical problem is solved from a practitioner's stance. Some of the above models could be classified as cognitive theories, since they model how a student solves and processes a statistical problem. None of the models surveyed in the literature mention anything about the 'constraints' (see Section 7.2

and Section 9.2.2.2) of working in a data-based environment, and how these affected statistical thinking and reasoning. Thus we would classify our framework as a theory which models practitioner behaviour. It may be useful for informing instruction and may serve as a link between some cognitive theory and some statistical practice (Lester, 1989).

# 11.3 Statistical Thinking, Statistical Reasoning, Statistical Literacy and Probabilistic Thinking

Statistical thinking, statistical reasoning and statistical literacy are often referred to in the literature yet there is no shared understanding of what these commonly used terms mean (Ben-Zvi and Garfield, personal communication, 1998). To this list I would add probabilistic thinking. All such perspectives are valid, for it is in the differences that more meaning is uncovered in studies of people's understanding of statistics. Our framework captures some of the meanings that different researchers attach to these terms and hence it could be useful for distinguishing, and for bringing together, such interrelationships.

Statistical reasoning in a data-based environment can be likened to statistical detective work (Shaughnessy et al., 1996; Biehler & Steinbring, 1991). The notion of statistical detective work and what that means has only been briefly referred to in the literature. Our framework attempts to uncover that meaning through an in-depth analysis of practitioners' thinking strategies during statistical investigation. The statistical detective way of reasoning is mainly catered for in the framework in the 'interrogative cycle', 'transnumeration', 'seeking explanations' and the 'integration of statistical and context knowledge'. The importance of data creation (Hancock et al., 1992) is clearly emphasised in our framework under 'problem', 'plan' and 'data' in the 'investigative cycle' dimension. We have identified the 'problem', 'plan' and 'data' stages, as neglected areas of statistical modelling and teaching. Also we have identified and described a subcategory of 'transnumeration', as the dynamic process of capturing relevant measurements from the system, which emphasises this domain of data creation. The objectification of knowledge is seen by Hancock et al. as an especially important reasoning process in statistics. This is catered for in our framework in three dimensions. The first type of objectification of knowledge is validation of knowledge. This type appears as the components: 'recognition of the need for data'; 'transnumeration'; and 'measurement' with its attendant data structures. The other type of objectification of knowledge is the statistical reasoning whereby evidence is weighed up. This is catered for in our framework in the 'interrogative cycle' especially in the 'criticise' phase.

In the area of graphicacy reasoning, Curcio's (1987) three components of reading the data, reading between the data and reading beyond the data, are incorporated into the

'interrogative cycle' under the 'interpret' mode and under 'reasoning with statistical models'. Hancock et al. also state that students find it difficult to reason about a group propensity, a fundamental skill in reasoning with statistical models, preferring instead to reason about individual cases. They believe this is a developmental problem. A complementary idea arising from the findings from my research is that students tend to give individual-based reasons as 'explanations for variation' such as age, depression (risk factors) etc., rather than system explanations, for specific contexts. This propensity of students to give individual-based explanations may be linked to Hancock et al.'s findings on the tendency of students to reason about individual cases, or may be linked to teaching approaches, or may be linked to a lack of context knowledge of the situation. Biehler (1994b) argues strongly that students must be able to give individual and system causes, as both types of causes will reveal more about the data being considered. In Chapter 10 our research and framework demonstrate, under the 'variation-context' criterion that there should be a positing of both types of explanation. Hancock et al. (1992) and Shaughnessy (1997) consider that interpreting graphs is different from constructing graphs, and using them as an analytical tool to reveal information in the problem. In our framework these concerns are identified in the 'transnumeration' type of thinking.

Landwehr et al.'s (1995) list for statistical literacy is at a more detailed level in some areas than that covered by our framework. Taking each aspect in the list I will demonstrate how each is related to elements in our framework: (1) The number sense in context could be categorised under 'integration of statistical and context knowledge'; (2) an understanding of variables, whether it is in the sense of conceptualising the idea of a variable, or understanding a particular variable, is related to 'seeking explanations' and 'transnumeration'; (3) an ability to interpret tables and graphs relates to the 'interrogative cycle' and 'reasoning with statistical models'; (4) knowledge of how a statistical study is planned is part of the 'investigative cycle' dimension; and (5) an understanding of how probability relates to statistics is related to how 'statistical models' address issues arising from 'variation'. Hawkins (1996) described statistical literacy as requiring skills in summarising and representing information for others, as well as the ability to operate in a non-deterministic environment. These are skills required for 'variation' and 'transnumeration'.

Another area of statistical literacy is the interpretation and evaluation of statistical reports, which is considered by Gal et al. (1995) to require the operation of both a cognitive and a dispositional component. For the cognitive component a list of 'worry' questions is proposed as a means of aiding students to critique statistically based claims. In Chapter 10 I have demonstrated how our framework can be used to generate such a list. Also, the dispositional component is catered for in our framework. Using the framework as a basis for the development of assessment criteria for interpretive skills, as in Chapter 10, may

partially answer Friel et al.'s (1997) concerns about having a clear idea of what educators mean by sound statistical reasoning, and may aid the development of better tools for making judgements about statistical reports. Gal et al.'s (1995) and Watson's (1997) preconditions for students being able to judge reports are the comprehension of statistical terminology and displays which, on our framework, could be categorised under 'statistical knowledge base' and 'reasoning with statistical models'.

Biehler (1994b) argues eloquently for teaching to acknowledge the dual nature of thinking in statistics. These two cultures of thinking are probabilistic thinking and deterministic thinking, which appear on the framework under the 'types of thinking' dimension in the categories of 'seeking explanations', 'variation' and 'reasoning with statistical models'. Whilst agreeing that these two ways of thinking are fundamental to statistics, this research demonstrates that there are multidimensional ways of thinking in statistics, and all may need to be developed in students.

In the first exploratory study we concluded that we needed to develop students' probabilistic and deterministic thinking, and to make students aware of their thinking. However the question arises as to our definition of probabilistic thinking. On reflection, it is used in the first exploratory study as thinking about variation as randomly generated. That is, there is an assumption when looking at data that there is an underlying theoretical distribution based on probability. For example, in a time series plot for quality control, or in fitting a regression line to data, the inherent variation, for modelling purposes, may assume an underlying normal distribution. The construction of statistical models for prediction, explanation and control is based on modelling and quantifying variation, where the quantifying of the variation is idealised as a probability distribution. In enumerative studies, a random sample is taken from a population and then the estimates for the population are based on underlying theoretical probability distributions. From the variation perspective, no two random samples from a population will ever give the same results, and this must be accounted for when making statements about the data themselves, or when generalising.

However there is another type of probabilistic thinking that my research has not explored. Conditional probabilities, relative risk, odds ratios, combinatoric reasoning, the calculation of probabilities from data and from random devices, frequentist and Bayesian probabilities and so on incur probabilistic thinking. Hence the few tasks on random devices, which I gave to the students, are not considered to be sufficient to make any judgements, or conclusions, about the types of thinking prompted in the area of probability. Thus, for this particular framework, probabilistic thinking arises from thinking about 'variation', and in 'reasoning with statistical models' in statistics. In summary, there appears to be differences and similarities in the definitions and interrelationships of statistical reasoning, statistical thinking and statistical literacy. However all appear to be addressing elements of the same broad reality and we see no contradiction among them. As a first attempt at the distinction among these terms we would hazard that: statistical reasoning is synthesising a chain of ideas into a logical argument using statistics; statistical thinking is thought processes that are triggered during data-based enquiry to solve a practical problem, and this includes statistical reasoning; and statistical literacy is the ability to read, understand and interpret statistically-based information, and to participate in statistical discourse which also requires statistical reasoning.

# **11.4 Misconceptions**

Tversky and Kahneman (1982) laid down the foundations for much of the subsequent research into probabilistic misconceptions. My research is not in this area. However they identified such misconceptions as, failure to take statistical variation into account, particularly in small samples, and misconceptions arising due to the context of the presented problem, and due to the imagination of the subject. Our framework gives prominence to 'variation' type thinking, and to the importance of 'imagination' and 'context knowledge', in the statistical process.

The primacy effect identified by Fischbein (1987) is confirmed in this research, though we have elaborated on the definition considerably. For this particular area of application we have defined it as the first-in-the-field effect, or precedent effect. The psychology-ofmeasurement effect that we identified is not referred to in the body of literature surveyed. We suggest statistics educators should be aware of this effect, and perhaps adapt teaching approaches and thinking tools accordingly. This identified psychology-of-measurement effect is indirectly alluded to by Fischbein when he refers to the characteristics of good thinking tools. He defines a good reasoning model as one which is intellectually acceptable and manipulable. Our findings indicate people prefer to relate to the variables that they themselves have measured. This gives a strong signal that we need to develop better models with which to reason and communicate, such as those models developed in brain imaging. Fischbein would regard statistical tools such as boxplots as imperfect mediums since student misconceptions have been identified in use and interpretation (Biehler, 1996). My research would suggest that either the tool should be changed for ease of communication, and/or the focus of instruction should change, through the drawing of attention to such elements as 'variation' and to such processes as the 'interrogative cycle' phases. Whatever the solution, our framework gives an indication of areas that need improvement.

The list of misconceptions identified by Landwehr (1989, cited in Shaughnessy 1992), considers the misunderstandings people have about variation. These misconceptions are at a more detailed level than that covered by our framework. Batanero et al. (1994) report on numerous misconceptions when people reason with statistical models. Students' ability to reason with statistical models is a category of thinking that needs to be analysed in depth, in order to understand or gain more knowledge about their reasoning processes. The specific misconceptions identified in such research may have implications for teaching, and/or may indicate a lack of consistency existing within the teaching of reasoning with statistical models. We suspect that some key elements such as 'variation', 'transnumeration' and 'measurement notions' are not prominent in most teaching approaches. In order to overcome misconceptions, it is suggested by Hake (1987) that students become engaged in an active dialogue with themselves, the data and other students. The 'interrogative cycle' could act as a springboard for the development of such teaching approaches, at a prescriptive level.

# **11.5 Instruction**

The research on successful problem solving points to some important factors such as: creation of a rich set of examples in the knowledge base; organised subject and context knowledge base; ability to recognise the applicability of known techniques to a new situation; affects and socio-cultural influences; learning modes and environments; and metacognition. From my research findings I conclude that these factors would also apply to statistical problem solving. Our framework incorporates these factors but does not suggest how a student should cognitively organise subject knowledge, or how a student should recognise when to use a particular technique. This is beyond the scope of my research question. Our framework contributes an overall perspective on facets educators must develop in students' thinking during the teaching process, and relates this thinking specifically to statistics.

Our framework should help instruction in that it provides some key elements on which to base the curriculum, teaching and assessment. It exposes the idea of problem formulation for the purpose of explanation, prediction and control. This should aid the process of 'convergence to an answer', which Hancock et al. (1992) identified as lacking in instruction in the American classroom. These key elements may also explicate the common thinking processes in inference and EDA, which Biehler (1994b) notes lack interconnection in instruction. Other authors (e.g. Singer and Willett, 1993; Lajoie et al., 1995) promulgate the notion of authentic practice in the classroom community. The findings in my research may be helpful in teaching since there has been an attempt to

elucidate practitioner's knowledge, and our framework gives some guidance on the dimensions of statistics that should be drawn to students' attention.

Borovcnik and Bentz (1991) and Falk and Konold (1991) believe that instruction should start with student's intuitions, which are easily accessible. My research reinforced this belief. The concepts of 'variation', and the 'seeking of explanations' in a statistical situation, need to be addressed in the teaching process. Chapter 9 gives some ideas on how we might think about such aspects, and strategies for dealing with them. This teaching approach should set up stronger links between deterministic thinking and probabilistic thinking, and reinforce these complementary ways of thinking in statistics. A case is also made for developing more tools to aid the thinking process. However, in Konold's (1994) experience, the creation of a thinking tool for developing notions of variation in significance testing, is insufficient to change students to a probabilistic way of thinking. This suggests that the elements identified in the 'types of thinking' dimension must become all pervasive in statistics teaching.

# 11.6 Statisticians' Concerns

Amongst some statisticians (Chambers, 1993; Bartholomew, 1995; Wild, 1994) there is the realisation that applied statistics must broaden its domain in order to mature as a discipline. Our framework acknowledges this position to include the whole of the investigative cycle. The aim of statistical investigation is, we believe, the gaining of knowledge and understanding in the context sphere. Barabba (1991) widened the sphere of statistical investigation even further than this to include the users of such knowledge. Thus the framework, in Barabba's terminology, is in his 'information-producing domain' but does not extend to his 'information-using domain'.

The quality management approach to statistical thinking is promoted in our framework under 'variation', 'seeking explanations' and the 'recognition of the need for data' to guide actions (e.g. Hare et al., 1995; Snee, 1990). In Chapter 9 ideas of quality management about developing tools to aid statistical thinking are discussed. In particular Chapter 9 discusses the connections amongst causation, variation, randomness and statistical models, extending, we believe, the quality management ideas further and deeper and generating ideas for all fields that use statistics. In deference to Cobb and Moore (1997), our framework and research findings could be described as primitive for a theory of exploration and interpretation. Nevertheless the framework is an attempt at providing some structure for complex thinking processes, and complex systems such as social science and epidemiology.

The nature of statistical thinking as described by some statisticians (e.g. Moore, 1990; Ullman, 1995; Britz et al., 1997; Sylwester, 1993; Cobb and Moore, 1997; Mallows, 1998) has been incorporated into our framework and is in consonance with my research findings. We believe that we have developed a more comprehensive account of statistical thinking than has been articulated up to June 1998. Our framework and research could help educators in knowing how to develop statistical thinking, and in knowing what characterises statistical thinking.

#### **11.7 Historical Perspective**

The emergence of statistical thinking during the last century appears to have been based on four main factors. The first factor is a fundamental realisation that it is necessary to gather and analyse data for the acquisition of knowledge about a situation. The basis to this factor is recognition that knowledge can be based on investigation, using an empirical scientific method. The second factor is a recognition that statistical models can be used to model and predict group human behaviour. Thus an interplay between the statistical model and the real situation resulted in a shift of thinking to include a non-deterministic view of reality. The third factor is the application of mathematical models to a variety of domains, resulting in new ways of thinking, perceiving and interpreting, in the statistics discipline. For example, these new ways of thinking occurred when mathematical error models were used by Quetelet in the social science field, and Galton in the biological sciences, and consequently became reinterpreted as variation or chance statistical models. The fourth factor is the development of new tools for analysis, arising from the new situations where statistics was being applied. These new tools helped to aid the development of statistical thinking. Statistical thinking appears to have arisen from a context-knowledge base interacting with a mathematical-(statistical)-knowledge base, with the resultant synthesis producing new ways of modelling and perceiving the world.

In evaluating our framework from an historical perspective, we can see that 'reasoning with statistical models' in a broad range of contexts promotes statistical thinking. Also the 'recognition of the need for data', an understanding of 'variation' and 'synthesising context knowledge and statistical knowledge', are some of the fundamental types of thinking. The framework identifies the use of techniques as a type of thinking. Quetelet applied the normal distribution to the 'average man' essentially by recognising the applicability of such a distribution and using it in an innovative way. He thus promoted the development of statistical thinking. Using problem solving tools for aiding the development of statistical thinking not only recognised the indeterministic nature of certain individual phenomena, but also the underlying patterns and causal relationships.

The framework acknowledges the key roles these two ideas play through the categories of 'variation' and 'seeking explanations'. Although 'transnumeration' is not promoted as a key element by historical researchers, inklings of it can be found in Quetelet's 'average man' distributions, which fostered the idea that a central limit effect was at work. Thus through changing the way data are represented, deeper understandings of the data and the real system are revealed.

#### 11.8 The Framework and Further Research

Our framework is constructed around the empirical enquiry cycle for statistical investigations. Therefore the framework does not address how probabilities behave, and people's understanding of how random models behave. Research on probability thinking could result in a framework that would be an improvement on the one we have proposed for statistical thinking, since statistics makes extensive use of probability models. Also there is the question of applied and theoretical statistics. This framework is based in applied statistics. A theoretical statistical perspective may modify our framework. Another area to consider for extending the framework, is going beyond the empirical enquiry domain to the domain where people use and act on these empirical findings.

We have attempted to characterise some elements of statistical thinking by conducting an analysis of the discipline itself. Therefore it has been based from one perspective. Also our framework for statistical thinking has been constructed without reference to the literature on mathematical thinking, or to the literature on developing general and critical thinking skills. In order to extend this framework further from these initial beginnings, research should be done from other perspectives to find out the differences and similarities, and to determine whether statistics is indeed an independent intellectual method (Moore, 1990). Our framework for statistical thinking would be enriched with research from multiple perspectives such as humanistic psychology and critical pedagogy. Also, research using different tasks to the ones used in the first and second exploratory studies, and research using people from different cultural backgrounds, would extend and enhance our framework.

There is a need for research on students at all age levels, and on statisticians, while they in the process of conducting a statistical investigation. My research obtained reflections on the approaches undertaken. The viability and validity of this framework could then be ascertained and developed further at the macro-level. The framework could also be used to assess students' thinking in the manner carried out in Chapter 8, and used to develop and analyse a variety of assessment tasks by determining the type of thinking stimulated.

From our framework and research findings a number of thinking tools could be developed for the 'investigative cycle'. These tools could be developed for students at all age levels, such as the one described in Section 9.4. These thinking tools could then be evaluated to determine whether they improve and develop students' thinking. Similar thinking tools for the 'interrogative cycle' could be developed and tested. In Chapter 10 a first attempt was made at developing a list of questions to ask when judging statistically based claims. This list could be refined and tested further for tertiary students. It could then be modified for use at different school levels, and tested for applicability and whether it will aid the development of such skills.

The curriculum, and hence teaching and assessment, could be constructed using our framework as a fundamental base. Furthermore, the framework could be used for suggesting research areas that need to be developed. For example there is a small body of research on reasoning with statistical models, yet little research in any of the other areas.

# 11.9 Conclusion

My research question was: "What are some characteristics of statistical thinking, at the macro-level, in the domain of the empirical enquiry cycle from problem formulation to conclusions?" As a result of engaging in an ongoing interpretive process of four exploratory studies, involving the interviewing of statistics students and statisticians, a four-dimensional statistical thinking framework was developed. We believe that this theoretical framework, grounded in data and the literature, elucidates some characteristics of statistical thinking and sheds new light on its nature. The framework, and the resultant discussion emanating from the research findings, have been an attempt to make explicit what has largely been implicit knowledge, gained through experience in a data-based environment. This theory will never replace experience but it may provide a base for the development of more scaffolding for students entering the statistical community.