Chapter Eight
The Fourth Exploratory Study : Towards Characterising Statistical Thinking

The themes and categories extracted from the statisticians' interviews formed, for us, an initial characterisation of statistical thinking. Using these abstractions, a phrase-by-phrase analysis of the project-student interviews in the fourth exploratory study resulted in a four-dimensional codification system to describe their statistical thinking processes. The evolvement of this system, and its subsequent testing on the first and second exploratory studies' student interviews, are discussed in this chapter.

8.1 Introduction

The positioning of this chapter posed a dilemma since Chapter 9 was being written concurrently and was being continually reconstructed as new insights emerged from the fourth exploratory study data. The third exploratory study had furnished and stimulated ideas about the need to develop thinking tools for statistical investigative work, about dispositions and about inherently statistical ways of thinking. However the interrogative cycle and the description of thinking simultaneously in four dimensions did not crystallise until a thorough analysis of the fourth exploratory study data had been undertaken. Furthermore this fourth exploratory study data developed not only further aspects of the model (see Fig. 9.1) but also provided some model verification. Thus this chapter, which describes the evolvement of the four-dimensional framework into a coherent organised form, should be read in conjunction with Chapter 9.

In the fourth exploratory study four female third year project-students and one first year male project-student were interviewed. Three were majoring in statistics, one was currently enrolled in one Stage 3 paper and the first year project-student was enrolled in Stage 1 statistics. All were in the age range 17-24, had a background of Form 7 mathematics, and could be classified as students. The criteria for selection were that the students were: (1) willing to participate; (2) currently undertaking a statistical project of their own; and (3) seemed to offer the opportunity for me to learn. It should be noted that the students who volunteered were the project team leaders. None of these students were known to me.

Before the interview, I listened to a presentation of each project. The students were individually interviewed in depth. Broad questions, following a semi-structured protocol (see Appendix Four) based on the statistical enquiry empirical cycle, were asked. The interviews were approximately one hour long. The questions were presented orally and were audio-taped. Unplanned probes were used in order to clarify the perspectives of the subjects. The transcript of the interview and the analysis and interpretation of her comments was presented to and subsequently corroborated by Beth (see Section 8.2). The other students were given the opportunity to comment.

From all the interviews in the four exploratory studies, a framework was created. This framework is fully discussed in the next chapter. In order to understand how this final framework was developed using the fourth exploratory study, the process and analysis are explained. First I demonstrate an initial characterisation of thinking processes using a statistician's excerpt from a transcript in Section 8.2. I then give, in Section 8.3, illustrations from the project-student interviews of how the final characterisation of statistical thinking was obtained. Finally in Section 8.4 this characterisation is tested on the student interviews from the first two exploratory studies.

8.2 Initial Characterisation

From a cross-analysis of the statisticians' interviews (see Appendix Three) the following emerging characteristics of statistical thinking were proposed.

Statistical thinking is the *integration of statistical and real problem understanding*. Certain elements underpin and/or facilitate it. For example:

- interconnecting processes
- constructing and reasoning from models
- understanding and dealing with variation
- seeking explanations
- transnumerating (a coined word - see previous chapter)
- interrogating constantly (including imagining)
- encapsulating complexity
- acknowledging and dealing with limitations

For this thinking to occur the statistician must interact with the problem situation. This means that the statistician will bring *dispositions*, and the environment of the problem situation will bring *constraints*, that will impinge on the statistical thinking.

At first the project-student interviews were analysed from the perspective of capturing more ideas on the thinking behind a statistical investigation, that the statisticians'

interviews had not raised. Secondly, the interviews were analysed in detail, using the common themes from the cross-analysis of the statisticians' interviews. Eventually subcategories of the themes emerged. For example, it became evident that variation had the subcategories of noticing, acknowledging and dealing with variation. A re-evaluation of the statisticians interviews reinforced that these themes and categories were emerging and could be identified (Pfannkuch & Wild, 1998a).

An abbreviated transcript of one of Market's stories is given as an illustration. Phrases are numbered for future reference.

Market's Story

"We did another job, for the City Council and that was measuring people's attitudes towards the recycling scheme (1), both attitudes and participation and volumes, and what we were trying to do is get an understanding why (2), maybe, certain suburbs . . . actually had lower levels of participation in the recycling scheme (3) . . . we did that by conducting a telephone survey (4) asking them what they recycled, the volumes they recycled, how often they recycled and then a lot of questions relating to their attitudes towards recycling and then demographics about the household (5), . . . got the data in and we analysed it and it's mostly sensible (6) and it actually agrees with what the council thought about what suburbs are good at recycling and what suburbs are bad at recycling (7) and this showed sort of the . . . you know, the yuppie suburbs like [a named high socio-economic area] are really the best at recycling and the suburbs like, you know, [a named low socio-economic area] aren't really so good at recycling (8) . . . And that was the assumption (9) . . . our clients [assumption] and we had their understanding of what was happening (10). But, it turned out to be one that was completely wrong (11) and it took us to look at the data, look at the numbers and to think about them and realise that we had this assumption completely wrong (12), . . . we looked at what sorts of things were being recycled in the different suburbs (13) and the way they were being measured (14) . . . Our client was measuring recyclings based on weight (15). It turned out that people in [the high socio-economic area] looked like they were better recyclers, simply because they were recycling heavier things, drinking wine [from glass bottles], . . . whereas people in [the low socio-economic area] were recycling plastic drink bottles (16) . . . and when you realise that you see that there's very little difference between the suburbs in terms of taking part in the recycling scheme and that really is a sensible interpretation of the data (17). . . . very near the end of the process . . . we were sitting down just talking and thinking (18) about how we would write the report and put the presentation together (19) and it sort of just dawned on us, we've been doing this thing all wrong . . . but yet at the time I had the sense that we might have missed it."

Discussion on Market's Story

Market's story will now be used as a basis for describing some elements of statistical thinking that were emerging, and could be identified from the interviews. Note that (not incl.) means (discussed in the full story but not included in the excerpted parts of this story).

At the beginning of a project, understanding the dynamics of the system in which the problem is set is necessary for the statistician to build up a mental model, or picture, of the *interconnections* in a system. This includes the understanding of how people operate in the system, and the clients' perception of the problem and system. To obtain this understanding, *interrogation* of people in the system and *interrogation* of the client occurs, as well as the observing of, and noticing *variation* in the system, and the seeking of alternative *explanations* for the phenomenon under study. Before Market could produce a survey on recycling, she needed to build up a model of how the recycling scheme worked, and to build up knowledge about recycling and the psychology of the people in the system (not incl.). At (1) she had made these *interconnections*, while at (2) she was seeking an *explanation* for the noticed *variation* (3). Once the system was understood and the problem *encapsulated*, measurement issues arise as to how to capture data from the system. *Transnumeration* occurs when measurements that reflect and capture the important elements from the real system are used at (5). The data collection process (4) by telephone was influenced by a cost *constraint* (not incl.). *Variation* was acknowledged and was dealt with in the design of the questions in the survey - that is, variation with regard to recycling and to the psychology of people being surveyed (not incl.). Despite this, part of a particular group's data still had to be discarded as a result of *interrogating* the data for reasonableness, and of recognising the *limitations* of what can be captured by measurement (not incl.).

At (6), as the data are being analysed, there is an *interrogation* process whereby with an internal dialogue, Market evaluates and judges, with reference to what she knows about the real situation. She also checks externally (7) with the clients as to whether the analysis is reasonable and sensible in terms of their understanding of the real situation. At (8), *transnumeration* occurs when the data are converted from summary statistics to a word interpretation form in order to facilitate communication. However a *primacy effect* blinkered Market to other possible interpretations of the data. At (9) and (10) the *constraints* of beliefs and expectations from the self and from the environment in which the problem is set resulted in a wrong assumption. At (11), *encapsulation* of an aspect of the analysis is possible through *interrogating* the data, seeking another *explanation* and noticing *variation* (conveyed implicitly in phrase 12) through *transnumeration* (13). Here, transnumeration is used, in the sense that data are looked at from many perspectives through reclassification, multiple graphs and transformations of the data. The

interrogation at (14) is an example of how, in each phase of the empirical cycle, a mapping to the real system and statistical system is engaged in, as well as looking forwards and backwards in the cycle in order to check the integrity of each phase. According to the statisticians interviewed it is crucial to check that measurements in the statistical system adequately capture crucial elements in the real system (*transnumeration*) which is demonstrated at (15). At (16) there is now an *explanation* for the *variation* and finally at (17), an *encapsulation* about how to interpret the data is determined. The importance of *interrogation* (18) is underlined at the final *transnumeration* phase (19) of communicating the data in a form that is understandable to the client (not incl.).

8.3 Final Characterisation

From the analysis of the statisticians' interviews the PPDAC cycle can be categorised into ten main stages (see Appendix Three). Each stage has further subcategories, one of which is exemplified in Section 9.4 of the next chapter. Some of the issues that should be considered at each stage are presented in the previous chapter. The ten main stages identified are:

1. Problem: understanding the dynamics of the system
2. Problem: defining the problem
3. Plan: measurement issues
4. Plan: design issues
5. Plan: data collection issues
6. Data: data issues
7. Analysis: planned analysis and modifications
8. Analysis: EDA
9. Conclusion: interpretation
10. Conclusion: communication.

At first I categorised and sub-categorised each phrase of the project- students transcripts with the classifications arising from the cross-analysis of the statisticians' interviews (cf Market's story in Section 8.2). However, inconsistent repeat codifications of the transcripts revealed problems with this categorisation system. Furthermore the project students were either describing their actions or their thinking. A rethink on the categorisation of the project-student interview data produced the realisation that modes of thinking, types of thinking and disposition should be separated. It became evident that the data should be characterised from four perspectives: PPDAC stage; interrogative cycle mode; type of thinking; and disposition. A cross-classification of the data made sense as one phrase in the transcript could belong in several dimensions. The mode of the thinking was best characterised by an interrogative cycle which was a running commentary on the thought processes that were being activated as the project was in progress. The

interrogative cycle had the five modes of seek, generate, interpret, criticise and judge and each of these modes had further subcategories (see Appendix Four). These are fully described in the next chapter. The types of thinking could be characterised as that thinking which was inherently statistical when working in a data-based environment, and that thinking which had the generic characteristics common to any investigation. The disposition was the attitude that the thinkers brought to the problem, and affected how they dealt with that problem. The PPDAC stage identified the particular phase of the empirical enquiry cycle to which the student was referring.

Examples of this detailed analysis follow. The modes of thinking cover the smallest phrases in the conversation whereas the types of thinking and dispositions cover larger phrases. Therefore it is not considered necessary to fill every box. Because the students are reflecting on their thinking processes, parts of the interrogative cycle are not explicit but can be implied as part of the internal thinking processes that are not articulated. However, over the hours of interviews analysed, all parts were observed.

8.3.1 Examples of Analysis

In these examples the main categories are underlined for the investigative stage, the interrogative mode and for the type of thinking. Where appropriate the subcategories are italicised.

Beth

Beth was part of a team of project students who were originally required to set one target supply time for the distribution department of a manufacturing company. The workers, who were going around to collect the goods to make up a customer order, could then aim to have the order completed within a certain time limit. Beth's project team were given nine months data consisting of packing slips which recorded the time the customer ordered, the time the order was completed by the distribution department, and the number and names of the items.

Beth Excerpt One

In the following excerpt (Table 8.1) Beth is talking about checking the data for reasonableness. The PPDAC stage is Data: data issues.

Table 8.1 Four-dimensional Analysis: Beth Excerpt One

Beth Excerpt One	Interrogative Mode	Type of Thinking	Disposition
There were one or two instances where we sat down and went "this looks way way out."	Criticising by evaluating *internally against her context knowledge*	*Noticing* variation	Scepticism Being observant
For example, normally the order would take less than twenty four hours. We had one order taking more than a week.	Interpreting the data and *connecting it to her context knowledge*	*Acknowledging* the variation	
And we sat down and went "this just looks real strange."	Criticising by evaluating *internally against her context knowledge*		
So we went back to the company and said "would this be reasonable?"	Criticising by evaluating *externally with her clients*	*Explaining* the variation or seeking explanations	Curiosity Scepticism
And they actually looked at that specific instance and said: "yeah, that would be, because that was one that had been back ordered and we had half completed the order but they were waiting for the rest of the stuff."	At this stage there is a judgement on the data *as to the reliability of the information*.	At this stage there is a synthesis of statistical and context knowledge	Perseverance

Beth Excerpt Two

Beth (Table 8.2) is talking about the analysis stage in which they were aiming to produce one target time as a performance measure for the distribution department. The PPDAC stage is Analysis: EDA since there was no pre-planned method of analysing the data.

Table 8.2 Four-dimensional Analysis: Beth Excerpt Two

Beth Excerpt Two	Interrogative Mode	Type of Thinking	Disposition
When we started looking at it and found there was a vast range, from five minutes to four hours.	Interpreting the data and *connecting it to the problem* under consideration	*Noticing* the variation in the statistical data	
And we sat there going, "okay, maybe this is not quite so easy,	Criticising the data and *anticipating problems* if produce one target time		
maybe we should talk to them."	Generating ideas about *how to proceed*		
And so we talked to them and said:	Criticising and evaluating the data *against an external source - the client*	Strategic planning	
"would it be reasonable to try and split it into two groups,	Generating a possibility *derived from working with the data*	Transnumerating the *data* would give a more realistic understanding of the data	
would that make sense with what you are doing?"	Criticising and evaluating this idea *against the external source, the client*		
Because we had known, we had found out right at the beginning, that there were different size orders depending on which season they were in. Whether it was the winter season with the dehumidifiers, or the air conditioning ones over the summer.	Criticising and evaluating *internally against her context knowledge* of the situation	*Noticing* the variation in the context of the problem and *suggesting an explanation* for the variation in context	Engagement
Then part way through we found, "this data is a bit too much."	A judgement on the *usefulness of the idea* to produce one target time begins to be made		
So that's when we went back to them and said "hey, can we split it?"	A check on this idea is accomplished by criticising it *against the external source, the client.* (Implicit in this statement is the generation of ideas about *how to proceed*.)	There is a synthesising of statistical knowledge and context knowledge based on variation and *explanation*. (Implicit is strategic thinking.)	

Beth Excerpt Three

Beth (Table 8.3) is now talking about part of the process of trying to work out how and where to split the data into groups. The PPDAC stage is Analysis: EDA. Even though standard confirmatory techniques were used the whole approach to the analysis was exploratory in nature.

Table 8.3 Four-dimensional Analysis: Beth Excerpt Three

Beth Excerpt Three	Interrogative Mode	Type of Thinking	Disposition
To start with, we put all the data into Excel	Judgement made to analyse the data using Excel, *on practicality of plan*	Drawing on techniques *from past experience*	
and we sat there and just plotted some cumulative frequency and some histograms . . .	Generating possibilities *derived from the data*		
To try and find a way in which we could break the data up into different groups.	Implicit, is that the plots would be interpreted and *connected to her context knowledge*	Transnumeration is occurring as she tries to find a way to model the *data*	
Something that was reasonable, and once we had found something that sort of worked,	She criticises and evaluates plots *internally against her statistical and context knowledge*		
we put the data into R.	Judgement at this stage to change the software used, *on practicality of plan*		
And used R analysis to get some confidence intervals and some restrictions and things like that.	Generating some *possibilities with the data*	Drawing on ideas of how data are modelled in statistics	
To see if we could get a much more clearer picture.	Interpreting the data generated and *connecting to her statistical and context knowledge*		
Excel, of course, the way we were doing it, we couldn't get exact numbers out of it, especially with calculating standard errors and things like that.	Reiterates her judgement about Excel. (Justifying of the judgement, *of the rightness of the encapsulation*.)		
So we thought "no". If we try putting it in R, that's known to give a lot more accurate figures.	Criticising and evaluating *internally* the judgement *based on her statistical knowledge*		
So we went through and put it into R and basically got a lot of summary statistics and some confidence intervals out of it.	The judgement of what she decided to do and *the rightness of the encapsulation*	Statistical modelling of data taking into account variation	

To further illustrate the interrogative-cycle modes of thinking, and types of thinking, excerpts from Beth's interview and the other interviews, are given.

Beth Excerpt Four

Well, firstly we really had to find out stuff about what the actual company was doing, to understand how they worked as a company, and then to try and understand the trail from when a customer rang the company, right to the end of the trail when the actual goods were delivered to them. So we had to understand that whole process before we could start working with that one little department. And so basically, there was a lot of background information, and the rest of it was just try the next bit, "okay, we can't do this because of this so . . ."

Beth's mode of thinking in this excerpt could be characterised as seeking *information from the system* in order to build up her context knowledge. The type of thinking is generic in that Beth is constructing a model of *how the system operates* in which the problem is embedded. The PPDAC stage is classified as Problem: understanding the dynamics of the system.

Beth Excerpt Five

When we were trying to analyse the data we were looking at, we split it into several groups. So, we first split it just overall and then we tried to split it, for example, by person, and seeing if each person had a different overall rate and things like that. But we found, for example, one of them we couldn't do, because the person had just left, so to try and analyse their data was quite difficult. And then of course we had some problems where they got one or two of the part-time workers coming in between that period we were doing. So there were odd bits here and there. It made it difficult to try and analyse some of that stuff a bit further. Therefore we couldn't get too much in depth. We had to stick to a lot more generalised bits and of course we were trying to find out, from the start, we were trying to find out which formulas we could use for this particular situation. And thank goodness we had lecturers, and they could help us out with those sorts of bits. Cause that was a big stumbling block - which formulas do we go for, what exactly are we looking at.

The modes of thinking in this excerpt are generating possibilities *derived from the data,* interpreting and *connecting,* then criticising and evaluating such possibilities based on a synthesis of context and statistical knowledge, and from that making a judgement on the *practicality of the plan.* There is an awareness of the *internal constraints* of the data, or of the *limitations* of what is possible in the analysis of the data. Thus a *distilling* and *discarding* process is operating. Beth also seeks information *internally* on which formula to use, is unsure, then *checks externally* with her lecturer. Sometimes the lecturer would

give them hints, or draw there attention to another possibility, or refer to a book such as particular chapters in their stage one textbook, or affirm what they were doing. The target type of thinking involved here is transnumeration of the *data to other forms, acknowledging* there is variation, *explaining the* variation or seeking explanations and finding a *statistical* model that will help in the solution of such a problem. The PPDAC stage is Analysis: EDA.

Beth Excerpt Six

You had to sit back and really think about what you were actually doing . . . We had to think "okay, now what does the company want?" And a lot of the things we'd done, they wouldn't understand, so we had to try and explain it to them in terms that made sense to them. So we had to stop and think, "okay, now we've got this, how do we rearrange it, or change it, or whatever, to get it into a form they can interpret and make use of?"

The mode of thinking illustrated in this excerpt is that in the criticising process there is a *monitoring of the purpose of the thinking* and an *internal check against the constraints* imposed by the clients involved in the problem situation. The type of thinking is around the idea that the statistical information must be transnumerated to a form that will *communicate an understanding about the real system*. The PPDAC stage is identified as Conclusions: communication.

Beth Excerpt Seven

To start with, we'd only got four months worth of data because he had only given us one box instead of the two he meant to. And we looked and there was only four months worth of data and we sat there and went, "this probably isn't enough to cover the yearly fluctuations that do occur." So we got another five months worth of data. Unfortunately that was all that they had. We would have liked to have the last three months so that we had the whole year to cover the whole range of seasons, but unfortunately they didn't have quite that. But we did ask them, "do you have that extra three months?" because, for the yearly fluctuations it would be good to put that in. Unfortunately they didn't have that one.

The modes of thinking triggered in this excerpt are the seeking of information in the form of *data from the system,* and the criticising and evaluating *internally* of such information using a synthesis of statistical and context knowledge. In the last sentence there is a realisation and criticism of the *constraint* and *limitations* imposed by only having nine months data available. The type of thinking is based around variation ideas, and the *explanation* of that variation. Closely allied with the transnumeration *of the system* type of thinking, is the raison d'être of statistics, when Beth recognises the need for data to reflect

the whole system in order to undertake a statistical investigation. The PPDAC stage is Data: data issues.

Cilla

Cilla's project team carried out a survey on customer opinion for an investment company. She is talking about the writing up of the final report.

Cilla Excerpt One

[We] *were both aware that we had been working with them for so long we had formed our own personal opinions, and were very cautious about maybe putting our bias onto the data . . . I got the gut feeling that they didn't treat us with respect. . . . We just put, "it's interesting to note that your female clients appear to have given you lower grades. Now whether this is just a suggestion females are much more demanding or this is an area which needs to be addressed is unknown."*

The mode of thinking illustrated here is that in the criticising and evaluating phase, Cilla is *monitoring and taking cognisance of her beliefs and emotional response* when interpreting and communicating her findings. During her interview it is evident that the analysis of the data is influenced to some extent by the team's perception of the attitude of the company to them. This led to the transnumeration *of data* into particular categories. However they are aware that their *beliefs and assumptions* could be influencing them and thus *monitored their thinking*. The PPDAC stage is Conclusions: communication.

Cilla Excerpt Two

That was where the big problem came in. The general questions they were asking were very qualitative, but they wanted it given in a quantitative form. . . . As one guy would always say, "they wanted statistical validity", which is very hard to get with the questions they wanted asked, which is a big problem because we didn't really know how to go about it.

They said they wanted us to look at their services and we thought about it. Again we didn't have a lot of experience. We [thought], *"if we were in the clients' position, what would we want." Like, if you wanted to meet your adviser, you're going to have to get there somehow. "How can you make it as easy as possible to meet your adviser?* [By ensuring] *parking is close by . . ."*

[Then] *what we did, was, we held a focus group with a few of their clients to get a feel for what they thought was important. And then* [we] *designed the survey around that base and tried to number it.* [For example] *you either thought the service was poor or excellent*

. . . You had a category base so you could say: "X percent believe your service is adequate." [We did it] *that way rather than just leaving it as a touch and feeling sort of approach.*

In this excerpt, the generic strategic type of thinking is illustrated by Cilla's *planning* of a focus group meeting as a way of generating relevant questions to ask in the survey. Before the focus group meeting, her project team's mode of thinking is to brainstorm and generate possible question areas *by imagining* what a customer of this investment company would want in terms of services. Other types of peculiarly statistical thinking illustrated here, are *dealing* with variation of customer opinion, for example, poor to excellent. And *suggesting explanations*, such as quality of service for customer variation in their perception of the company, then transnumerating that idea *by capturing it in data form*. There are two PPDAC stages being described in this excerpt, the first being Problem: defining the problem while the second refers to the Plan: measurement issues stage. This illustrates the iterative process of the empirical enquiry cycle. In this case, there is a shuttling between these two stages since the discussion of measurement issues helps to clarify the problem and the clarification of the problem helps in determining what to measure.

Nia

Nia was involved in a project for a bank which wanted to know whether it could raise the price for safety deposit lockers. Nia is talking about how they analysed the data.

Nia Excerpt

We found out who thought it [the price] *was unreasonable. We pulled out these customers and tried to figure out why they thought it was unreasonable. "Was it that they were X* [a named ethnic group], *was it that they had a locker previously in another country, or was it just some kid who was ticking no because they thought they would just tick no for no one reason?" 'Cause we had a small minority of X, but they were the ones that said: . . . "I think the price is too high", so we came back to the bank saying that everyone else said it* [the price] *was,* [that is] *the average, most people said: "that's okay."* [But there was] *a small little lump at the strongly disagree area. We went back and saw that it was the X. Well that was something that they* [the bank] *wanted. If we hadn't have gone back then we would have said: "Go on, increase the price."* [That is, if] *we took the average,* [we would say increase the price] *and then they'd lose their core business because it's mainly X who want the deposit lockers.*

In Nia's excerpt the type of thinking illustrated is investigating and *explaining* the variation, and, through a synthesising of context and statistical knowledge, making a

judgement on *the most likely scenario,* and therefore what statistical data to use for this particular situation. Nia talked often about having to work out what is important and what is not. Here she has to *discard* the average, which as she says, would be the conventional solution. Instead she focussed on the opinion of X which is important and relevant to the solution of the problem. Here the context is a determining factor in the solution. We can also see *noticing, deducing, distilling and encapsulating* operating in this excerpt. Again the shuttling between the two PPDAC stages of Analysis: EDA and Conclusions: interpretation is observable in this extract.

Lee

Lee's project was on developing a system for monitoring the progress of clients in a counselling service. Because of the nature of the counselling service the project team members were unable to observe the system in action or gather data themselves.

Lee Excerpt One

We had our spreadsheet up there. [It was] *our second one because our first one was based on their* [the counselling service] *check sheet. We thought this* [the first spreadsheet] *is just going nowhere. And the second one that we did, we were getting towards it* [a solution], *but we hadn't quite stumbled upon the accounting idea. Then we thought, well, "what if we can look at who's coming in and look at where they go." That was when it all came together. It was also a matter of sitting down. Because going through you'd think, "this is it", and you'd go through the historical data and go "hey, here's an anomaly here." Then you have to work out something that will cater for that . . .*

The type of thinking illustrated in this excerpt is transnumeration or finding a way to *capture data from the system* that is useful and meaningful for that system. At the same time, the generic type of thinking for modelling *the system* is operationalised. In Lee's interview she gives more detail about how they transnumerated *the system*. She considers that this is one of the hardest parts to resolve. She feels that because she has a background in statistics she knows what will make good data, and also she looks at a system from a statistical perspective by searching for a way to capture data from it. It is through a synthesis of her statistical knowledge and her in-depth understanding of how the counselling programme operates that she is able to create a way of measuring and recording the operations within that system. In order to check that her transnumeration *of the system* works she has to *use historical data* to criticise and evaluate it. This quotation demonstrates further the iterative nature of the PPDAC cycle as Lee constantly cycles through the Problem: defining the problem, Plan: measurement issues, Analysis: EDA, and Conclusions: interpretation stages in an effort to find a way of capturing data that reflect the reality of the system.

Lee Excerpt Two

At that stage, we had gotten quite down on ourselves. Thinking we're not doing really well. We hadn't, we'd lost touch with them because they were collecting the historical data for us. Meantime, we're trying to do this [model the system], *and it was at the point where we thought, "oh man, we're not very good." But as soon as we presented it to them at a meeting and N just said, "this is fantastic", then things picked up again.*

This demonstrates the criticise category of the interrogative mode of thinking since Lee's project team floundered for quite a while on their own, and thus are *monitoring their ability* to solve this particular problem. They learnt from this particular episode to use other areas of the criticise and evaluate mode of thinking, by *talking to external sources* such as their *client or lecturer* to clarify their thinking, to seek extra information, and to check they are on track. This is a common learning theme amongst the student interviews. In the first part of the extract the PPDAC stage Plan: data collection issues is referred to while the second part illustrates an interim Conclusions: communication stage.

Ray

Ray was a biology student who did not believe a Council's explanation that borer had killed some mangroves. He believed it was other human factors such as a recent oil spill in a mangrove swamp that had been the cause.

Ray Excerpt

I wanted to just see if there was a support [evidence] *for other human activities affecting the health of them* [mangroves]. *Then you would expect, in the areas where there was a lot of human activity, that if it* [borer] *was affecting them* [mangroves] *then it would affect their health. So if I measured their health and then compared it to an area where there wasn't a lot of human activity: "Was there a difference in health?" And then by selecting swamps that were very similar in sediment composition, and depth, and water temperature, and rainfall, and things like that. Both* [swamps] *had major river systems coming into them. I mean almost every aspect of them that it was practical* [to have]. *We couldn't have them identical, but* [they were] *practically the same, except for the level of development of the land around and the way people use it. So therefore I could hope that any differences would be due to that.*

Ray started this project which is based on his judgement of the *relative plausibility of competing explanations* and of the *need for more research.* From this description we can see that the type of thinking revolves around *dealing* with variation in his experimental design and being able to provide an *explanation* for the variation and a way to validate the

explanation. In this excerpt the PPDAC stages Problem: defining the problem and Plan: design issues can be identified.

8.4 A Return to Exploratory Studies One and Two

In order to test further these proposed characteristics of statistical thinking, I looked at earlier interviews of students working on statistical tasks. I found that their thinking could be characterised using these four dimensions. Once they were classified it became easier to evaluate what was 'missing' from their thinking, and the classification also provided a way of analysing the type of task presented. The difference that was highlighted between the project students and students asked to do tasks, was disposition. The task students did not appear engaged or interested in the problem and therefore questions must be raised about the validity of deriving information from such tasks.

8.4.1 Exploratory Study Two Examples of Analysis

Isa

Isa Excerpt One

From exploratory study two, Isa's interview (Table 8.4) is used to illustrate how these dimensions are tested. This task given to him is in the PPDAC stage Analysis: EDA.

Map Question

Every year in New Zealand approximately seven children are born with a limb missing. Last year the children born with this abnormality were located in New Zealand as shown on the map (Fig. 8.1). What do you think?

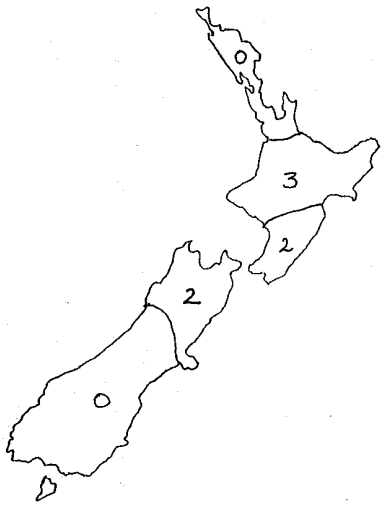

Figure 8.1 Map Question

Table 8.4 Four-dimensional Analysis: Isa Excerpt One

Isa Excerpt One	Interrogative Mode	Type of Thinking	Disposition
I would probably want to know why,	Seeking *information internally*		
in the central North Island three deformed children are born, and two are born in the south of the North Island.	Interpreting the data *by reading*	*Noticing* variation in the context	
Why is it in the North and South?	Interpreting the data further *by comparing*	*Seeking explanation* for the variation	
Though the immediate thing you think, is, the north of the North Island and the south of the South Island are pretty sparsely populated.	Seeking information *internally about his context knowledge* of the situation		
I don't know why. I can't say why, for that, for the reasons you would make me ask myself.	Criticising by *monitoring the purpose* of the question		General disposition is non-engagement
Agriculture is something that is spread right throughout New Zealand.	Generates a possibility *using context knowledge*	Suggests *context explanations* for the variation	
It's not like pesticides use is going to have any more effect. I don't think in the central North Island than in the north of the North Island - pesticides are used everywhere.	Criticises and evaluates this possibility *internally against his context knowledge*		
It's a bit of a difficult one, an odd one. I don't know. Central North Island I use it as an example, maybe, yes I don't know actually.	Judging the *usefulness of ideas* by deciding he can't go any further		
As far as hospitals and medical aid it's there.	Generates another possibility		
It's there also in the south of the North Island - Wellington.	Criticises and evaluates that possibility		
No I can't think of any other reasons for that.	Judges *usefulness of ideas* and decides to finish problem		

If Isa's response to this question is analysed, it can be seen that he does not use his statistical knowledge to think about the problem. Nor does he critique the problem in relation to: 'variation'; 'constructing a statistical model' of the situation based on population proportions - and the inherent 'transnumeration' in that process; and the 'transnumeration' of how the data are communicated. In the context sphere these facets are being operationalised. Consequently 'context knowledge and statistical knowledge' are not synthesised or integrated. The 'interrogative cycle' is activated only in the context sphere.

Isa Excerpt Two

Isa (Table 8.5) is referring to the following task in the PPDAC stage Conclusion: Interpretation.

Error Rate Question

In a firm in Wellington the management was concerned at the number of errors that office staff were making in transactions. The four office staff were audited every day over a month and the box-and whisker plots shown were obtained. If you were the manager and had been presented with this graph (Fig. 8.2) what would you think?

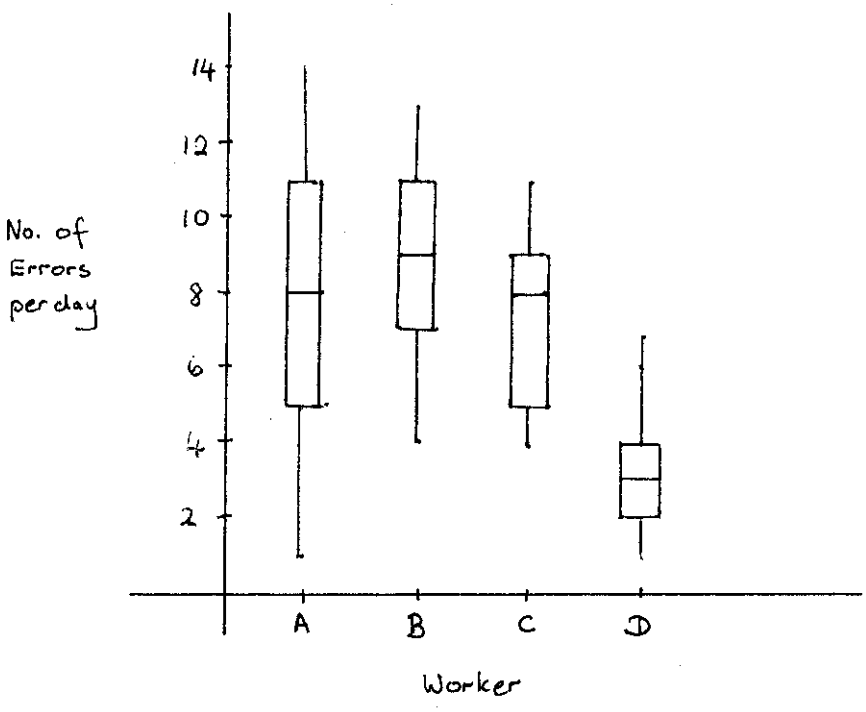

Figure 8.2 Error Rate Question

Table 8.5 Four-dimensional Analysis: Isa Excerpt Two

Isa Excerpt Two	Interrogative Mode	Type of Thinking	Disposition
I would start by thinking that worker A is either very erratic.	Interpreting the data by *connecting to the context*	*Noticing* the variation in the data and connects to context knowledge	
I mean he goes from 1-14 errors - yes, that's per day isn't it	Interpreting the graph by reading it		
I wouldn't say that Worker A was, I would say he's more than likely careless.	Interprets the data by *connecting to some context knowledge* and makes a judgement	Suggests context explanations for variation	
Because obviously there are times when he doesn't make errors. So he's probably a careless sort of person.	Judgement process in action on the *most likely of a set of possible scenarios.* (Judgement and justify.)		
Worker D seems to me to be a fairly cautious person.	Judgement on the *most likely of a set of possible scenarios* for Worker D		
Yes, Worker D - for tending to keep his errors to a minimum, a methodical type person.	Interprets and *connects to the context*	Suggests a context *explanation*	
And Worker B seems to be consistently making errors.	Interprets by *translating the graph information*		
I would probably want to know why.	Would seek *information from the system*		
Possibly that person is a Manager.	Generates a possibility		This comment would suggest he is not engaged in the problem
I would think that person is obviously doing a job that he shouldn't be.	Judgement in action on the *most likely of a set of possible scenarios*		
Worker C, also, I find makes - yes, he seems to make quite a few errors, but his mean is no higher than A.	Interpreting the data *by comparing*		
So I'd put a question mark over that, I'd want to question him.	Would seek *information from the system*		

Isa ended up by judging the workers from excellent to poor. In the 'interrogative cycle' the criticise part is missing, as he immediately jumps from 'interpret and connect', to 'judgement'. Nor does he proffer multiple explanations for interpreting the data, preferring instead to have one context explanation. The type of thinking is mainly in the context sphere. In relation to statistical knowledge he offers no explanation for, nor deals with 'variation' among Workers A, B and C as a group and the 'variation' between this group and Worker D. He does not critique the data on the grounds of 'transnumeration', nor on how the data are captured through measurement. Again there is no 'synthesis of statistical and context knowledge'. From the statisticians interviews it became obvious that data cannot be adequately interpreted unless all the phases of the 'PPDAC cycle' are understood. Thus if students do not have adequate knowledge of the data literature for the given tasks then we would suggest that their interpretation will be limited and their disposition towards the problem will be affected.

8.4.2 Exploratory Study One Examples of Analysis

Exploratory Study One is now used to further test these classifications.

Ebe

Ebe Excerpt One

In this problem Ebe (Table 8.7) is describing how she solved the following probability table problem which can be classified as the PPDAC stage Analysis: Planned.

A study categorized both alcohol intake (prior to pregnancy recognition) and smoking (during pregnancy) as "none", "moderate" or "heavy". The table below (Table 8.6) gives the probability of the given level of alcohol intake and smoking.

Table 8.6 Alcohol and smoking status for pregnant women

		Smoking			
		None	Moderate	Heavy	Total
Alcohol drinking	None	.232	.015	.024	.271
	Moderate	.314	.093	.122	.529
	Heavy	.127	.035	.038	.200
	TOTAL	.673	.143	.184	1

Given that a pregnant woman is a heavy drinker, the probability that she is a heavy smoker is:

(a) .2 x .184 (b) .038 / .2 (c) .038 / .184

(d) .2 + .184 - .038 (e) .024 + .122 + .127 + .035

(Question from Department of Mathematics and Statistics, 1993)

Table 8.7 Four-dimensional Analysis: Ebe Excerpt One

Ebe Excerpt One	Interrogative Mode	Type of Thinking	Disposition
Well, I just look at the question first, like here, given that a woman,	Seeks information from the *external problem source*		
which to me, in the "given that", in this case, is made as a category.	Seeks information *internally from similar problems in her statistical knowledge*	Techniques are recalled *from past experience* on similar problems	
Given that a pregnant woman is a heavy drinker the probability that she's a heavy smoker.	Seeks more information from the *external problem source*		
I look at that. I know it doesn't sort of make sense but I understand what I am saying.	Criticises by *monitoring the purpose of her thinking*		
So I looked at that, that was my heavy drinker, that's what I look at.	Interprets the table by *reading the category*		
And the probability that she is a heavy smoker, which is down there (points).	Interprets the table further *by reading* another category		
That's the result for a heavy drinker. You know the percentage and probability, and that's for the heavy smoker over there. That is how I got my answer.	Judges the answer, *the rightness of the encapsulation*, by seeking information *internally from a similar problem*	Techniques are recalled from *past experience on similar problems*	

If an attempt is made to classify this problem, it becomes apparent from the 'interrogative cycle' perspective that 'interpret and connect', and 'criticise and evaluate', using a 'synthesis of statistical and context knowledge', are non-existent. 'Variation', 'explanation', 'transnumeration', and 'constructing models' are not present which suggests that this proposed classification method could be used for such a task. In fact this task is not rich enough to display any of these characteristics.

The next two examples demonstrate different answers in different contexts. First Ebe (Table 8.8) is given a graph and asked how she would interpret it. This can be classified in the PPDAC stage Conclusions: Interpretation.

Ebe Excerpt Two

Test Results Question: A small class was given a test on arithmetic and the results were recorded. The same test was given a few weeks later. The box-and-whisker plots for both sets of results are shown (Fig. 8.3). Have the results changed much? If so, can you give any possible reasons?

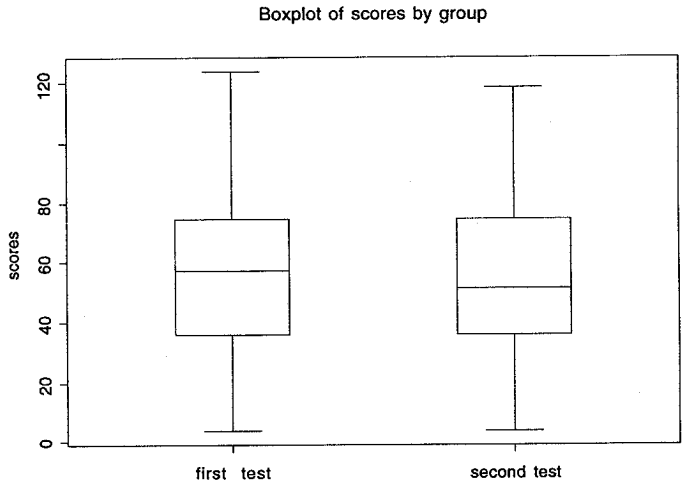

Figure 8.3 Test Results Question

Table 8.8 Four-dimensional Analysis: Ebe Excerpt Two

Ebe Excerpt Two	Interrogative Mode	Type of Thinking	Disposition
As I see these - it looks like - if it was the same test it should be higher.	Interprets the data and *connects it to the context*	*Noticing* the variation	
But it's not.	Criticises *internally against her context knowledge*		
So I'm not sure why. I don't know.	Makes a judgement that there may be a *need for more research*		
Lack of concentration. They've had the test before so they are a little bit bored about it or something like that. I'm sure there would be other reasons but I just can't think of any at the moment. Or it depends on how they feel, whether they are well or not, time of day.	She then thinks and generates some possible explanations *using her context knowledge*	Suggests *context explanations* for the variation	

Ebe Excerpt Three

Ebe (Table 8.9) is given some data on coin tossing and asked to think about how she would interpret this situation in the PPDAC stage Conclusions: Interpretation.

Coin Toss Question: A fair coin is tossed 50 times resulting in 27 heads. Two days later it is tossed again 50 times resulting in 30 heads. What do you think of these results?

Table 8.9 Four-dimensional Analysis: Ebe Excerpt Three

Ebe Excerpt Three	Interrogative Mode	Type of Thinking	Disposition
Well, I wouldn't be surprised about the results.	Judgement based on *conformance with both context-matter and statistical understandings*		
I think that on another day that's going to be different, could be less, could be more.	Criticising and evaluating this judgement *internally based on her statistical and context knowledge*	*Acknowledging* there is variation	
I mean, in a way, if I got 30 heads I'd like to keep tossing just to make sure.	Would seek more *information from the context*		
I would like to toss it probably more than 50 [times] to see . . . If it was a bigger sample, if that would average out to be fair, 'cause it doesn't seem like it's fair if you got 30 heads.	Criticising and evaluating *internally from her statistical and context knowledge*	*Dealing* with the variation Suggests a context *explanation*	

The above two examples illustrate that when a problem is in an unfamiliar context the student resorts to 'context knowledge' and does not use her 'statistical knowledge', whereas in the familiar context of coin tossing there is an 'integration of statistical and context knowledge'. In the former example, during the 'interrogative cycle', she does not 'criticise and evaluate' the possibilities that she has generated. The 'disposition' of Ebe perhaps can only be described as neutral.

8.5 Discussion

In the above analysis, problems of interpretation arise as the students do not articulate fully their thinking processes. Thus a partial model of their thinking is revealed and the categorisation is subjective on my part. Through making sense of what is being said by the students, I must conjecture the explicit in what is implicit. However, my supervisor, a statistician, independently agreed with the categorisation and thus the proposed categories seem viable. Another method used to decide whether the categories are viable was to give

the excerpts to six mathematics educators for comment and confirmation. Also Beth, a project student who was interviewed, was given this chapter and subsequently verified the proposed categories.

The analysis revealed the iterative nature of the interrogative cycle, and how the PPDAC cycle is composed of many such interrogative cycles. There appears to be a dynamic fluid interaction among all the modes of the interrogative cycle. Also, from this detailed analysis, a four-dimensional codification system emerged. The components of 'how one thinks' in statistics are categorised as dispositions, interrogative cycle modes, investigative PPDAC cycle stages, and types of thinking. These are fully discussed in Chapter 9. Since the analysis is based on project students reflecting on the thinking they have used in a particular investigation, I decided to test the proposed categories on the student interviews from the first and second exploratory studies, where the students are thinking in situ.

The implications arising from this analysis are that the proposed dimensional categories may be useful for determining what thinking a task is prompting, and what thinking a student is using. Another issue raised is the validity of interpretation of students' thinking when they have little or no context knowledge of the task, or no knowledge of the purpose of the task, or little need to be engaged personally with the task.

The framework discussed in the next chapter could be seen as a springboard for developing tools in teaching to promote statistical thinking, and for analysing tasks in relation to the statistical thinking dimensions.