Chapter Four
Research Design

> *"Qualitative research is like art where one engages and develops experience with a sense of meaning"* (Janesick, 1994, p. 209).

The research method I have used is an exploratory qualitative design. As little work in statistics education has been done on the nature and characteristics of statistical thinking the method is necessarily exploratory.

4.1 Research Design Assumptions

This research is concentrated on obtaining information from a few subjects in four exploratory studies. The assumptions behind the research design are:

- To learn about how people think they must be treated as people and therefore I must interact with them. The subjects must not be treated as faceless and asked to fill in a questionnaire (Fontana & Frey, 1994). It is better to collect data thoroughly, from an interview protocol with a few people, to uncover meaning (Smith, 1997) rather than to try to uncover meaning from decontextualised individuals.
- The choice of subjects will be made on the basis that they seem to offer me an opportunity to learn and advance my understanding of statistical thinking. A variety of subjects will be chosen from different backgrounds and experiences but they will not necessarily be representative of their group. The potential for learning is considered a superior criterion to representativeness (Stake, 1994).
- On the basis of ethical considerations, subjects will participate on a voluntary basis and therefore will demonstrate a self-selection bias. Presumably only the most confident people in statistics will volunteer to reveal their own thinking, particularly when they do not know me.
- A semi-structured interview protocol will mean that both I, as the interviewer, and the interviewee will be learning throughout the interaction. I will not be bound by pre-determined categories of measurement but will be free to search for meaningful categories and patterns (Adler & Adler, 1994). I will be an integral part of the research process and will be hypothesis generating and testing as well as having intuitive reactions in the field. I will be looking for meaning, and the perspectives of the subjects, during the interview process.

- The gauging of a subject's thinking processes in an *interview* will not ensure that all thinking is captured. Only a partial model of the reality will be revealed as the roles of the interviewer and interviewee are clearly defined and therefore not indicative of what will occur in a peer or other interaction.
- There will be **an ongoing analysis and interpretation of the data throughout** the exploratory studies. I recognise that the models used to understand the participants' thinking are subject to revision and restructuring (Lesh & Kelly, 1994) and are necessarily subjective.
- The theory building and generalisations will be inductively derived from the exploratory studies and grounded in the data. The theories and conclusions will be about these exploratory studies only. My research will be focussed on understanding the thinking of the participants not on making predictions. The analysis, that is the categories, themes and patterns will come from the data and will not be imposed before data collection (Strauss, 1987; Miles & Huberman, 1994).

4.2 Researcher Biases

As the researcher, I will play a pivotal role in the problem formulation, data collection process and interpretation of the findings. Therefore my own biases and ideology will necessarily be part of the process. However there should be recognition that there is no value-free or bias-free research design (Janesick, 1994). The biases identified by me are:

- I have an intrinsic dislike of gambling-type problems and competitive games and a preference for problems in a human context that will reveal more about the human condition and environment.
- I have a preference for looking for a global picture and searching for an understanding of the whole. I have a propensity for learning about how and why something works.
- As a mathematics secondary teacher for over sixteen years and a mathematics teacher trainer for five years I believe that a judgement of how a person thinks cannot be obtained from a paper assessment and that the only way to obtain a richer understanding of thinking is through talking to that person (Ahmed, 1987).
- Because of years of teaching and an innate dissatisfaction with rote learning I have an intrinsic interest in motivating students to learn to think and therefore a quest for understanding what that thinking is in statistics, and what teaching approaches should be used to develop that thinking.
- As the time line for the research is six years, and as I will be concurrently engaged in teaching statistics at the undergraduate level and supervising statistical projects

at the graduate level, my perspective will evolve through the research but, because of my heightened awareness of certain research issues, also through my teaching .

4.3 Methods for Overcoming Research Design Biases

Data Triangulation

A variety of data sources have been used in the study. One data source was from undergraduate male and female students with backgrounds ranging from: school leaver to people with established careers; Form 4 mathematics to Stage 2 mathematics and statistics; and age 18 to 50 years. Another data source was statisticians from a wide range of disciplines such as quality assurance, biology and forensic science and from a background where they were involved with projects as statisticians and as supervising teachers.

Investigator Triangulation

My supervisor and I independently categorised the data then reached a consensus through specifically checking and testing whether the analysis and interpretations of the data were traceable to and grounded in the data. The publication of two refereed papers (Pfannkuch & Brown, 1996; Pfannkuch, 1997b) from the study is considered to be another check on whether empirical assertions, the analysis, and discussion, seem logical and convincing to other people in the field. All the statisticians and one project student involved in the exploratory studies either corroborated or refuted the interpretation of the data pertaining to them.

Methodological Triangulation

The methods used in my research have involved undergraduate students: (1) reacting to or (2) providing solutions to given information; and (3) reflecting on a statistical project they had undertaken where they were in a position of creating information. These three aspects should be seen as different ways of obtaining information on thinking as the student is involved at a different level. The statisticians were involved at two different levels, either reflecting on their own statistical thinking or on their clients' or students' statistical thinking. These different ways would appear to provide multiple methods and perspectives in my research.

4.4 Ethics

Since this research involved the interviewing of subjects, the conduct of the studies including the research method, the aims and rationales of the studies, the method of data analysis, the participant information sheets and participant consent forms were submitted and approved by The University of Auckland Human Subjects Ethics Committee. The conduct of the studies, including the written consent of all participants, was in accordance with The University of Auckland policy. The University of Auckland policy also states that no-one will have access to the data except my supervisor and me and therefore this clause precludes the publication of any transcript in this thesis.

4.5 Research Design

Janesick's (1994) three stages to qualitative design are used as a framework to explain the research design. The three stages are: design decisions made at the beginning of the research; design decisions made throughout the research; design decisions made at the end of the research.

4.5.1 Design Decisions at the Beginning of the Research

4.5.1.1 Exploratory Study One

We held the opinion that a formal mathematical approach to teaching probability may serve as an obstacle to the development of statistical thinking. The first exploratory study was set up in 1994 to investigate the probabilistic learning in a small group of adult undergraduate students. The stated aims of the study were:

1. To provide directions for a future in-depth study.
2. To conjecture some factors regarding subjects' primary intuitions about probability and statistics.
3. To conjecture methods by which teaching can establish links between subjects' intuitions and the mathematical model.

In particular we sought answers to the following questions: (1) what understanding of variation do students have? (2) how do they think about probability in various contexts? and (3) what can be done to increase their understanding of variation and probability? The exploratory study consisted of three phases, each of which was intended to build up a more complete understanding of some students' thinking. Phase One consisted of individual interviews, designed to provide tentative answers to questions (1) and (2). On the basis of this first interview, a one day course (Phase Two) was designed to experiment with some teaching approaches, towards a possible answer to question (3).

Phase Three was a follow up interview in which apparent changes to understanding were investigated.

Subjects

The students (Table 4.1) selected for the study were all majoring in psychology for their degree and were currently enrolled in my stream of the first year statistics course. The criteria for selection were that the students would have previously established a working relationship with me, demonstrated a commitment to learning and seemed to offer me an opportunity to gain an insight into some students' thinking.

Table 4.1 First Exploratory Study Subject Data

Subject	Gender	Age Group	School Maths Level	Statistics Level	Personal Background
1	F	40+	F6	Enrolled Stage I	Technician
2	F	40+	F6	Enrolled Stage I	Professional
3	F	30-39	F4	Enrolled Stage I	Professional
4	F	30-39	F5	Enrolled Stage I	Semi-skilled
5	F	30-39	F4	Enrolled Stage I	Semi-skilled

The school levels in a New Zealand high school range from Form 3 (13-14 year olds) to Form 7 (17-18 year olds)

Phase One: First Interview

At the time of the interview the students had completed course work on 'Tools for Exploring Data' (includes numerical and graphical summaries) and "Probability' (includes simple probability, probability rules, conditional probability and statistical independence). The students were interviewed individually for about one hour. They were told that I was interested in their thinking and reasoning rather than in their getting the 'correct' answer. Consequently they were asked to think aloud during the interview. The questions were presented orally and on paper. Unplanned probes were used in order to clarify the students' thinking for me. The interviews were audio-taped. In order to put the students at ease the first set of items were typical course assessment items followed by non-standard items (see Appendix One).

Phase Two: The One-Day Course

Based on a literature search and my and my supervisor's interpretation of the interviews, the decision was made to focus the one day course on:

- deliberately thinking both deterministically and probabilistically about problems;
- experiencing experiments that reveal small samples are not representative of the population (representative used in the non-technical sense);
- experiencing probability based on models and real data;
- clarifying variation and chance.

Two weeks after the interviews a one-day, five-hour course was held (see Appendix One). Twice during the day students were asked to write down what they had learnt. At the same time I wrote down my impressions of the course.

The course was interactive, involved a lot of discussion, challenging of ideas, practical activities and simulations. Students were encouraged to think aloud and clarify their ideas. The instructional approach was based on these criteria (Konold, 1991): students (1) reveal their own beliefs first and then listen to the beliefs of others; (2) think about the problem deterministically and probabilistically; (3) observe through a hands-on simulation the results of a random process and communicate and discuss what they are thinking; (4) observe, through a computer simulation using *R* (Gentleman & Ihaka, 1994) software, the results of a random process from small to large samples, and communicate and discuss what they are thinking; (5) reflect on and evaluate what they originally thought in comparison to their observations from the data; (6) reconsider the dual modes of thinking probabilistically and deterministically for the explanation of variation. We believed that simulations would enable students to experience variation and hence strengthen their understanding. Criterion one was considered to be important as students need to reveal their intuitions and have a personal involvement in the task, and it is also a base on which the teacher can build.

Phase Three: The Follow-Up Interview

Three weeks after the one day course a second interview took place to further investigate students' statistical thinking, beliefs and intuitions. The procedure was similar to the first interview. The students were each interviewed for one hour. The questions were presented orally and/or on paper depending on the nature of the question (see Appendix One). Again the students were asked to 'think aloud' and unplanned probes were used.

Decision

The decision was then made to use a similar design approach in a second exploratory study with the teaching phase being a longer period of time.

4.5.2 Design Decisions made Throughout the Research

4.5.2.1 Exploratory Study Two

The second exploratory study was set up in 1995. The stated aims of the study were:

1. To provide further directions for future related studies.
2. To test a teaching method for establishing a concept of probability.
3. To conjecture further factors that enhance statistical thinking.

This study consisted of three phases, the intention being in the first and third phase to build up a more complete understanding of some students' thinking. Phase One consisted of individual interviews of six students. Phase Two was a five week course of 15 hours, of which 10 hours were in a lecture theatre and 5 hours in a computer laboratory The course was given to a Science Communication class of 60 Stage I students. Phase Three was a follow up interview of the same six students in order to see whether there had been a shift in statistical thinking.

Subjects

The students (Table 4.2) selected for the study were currently enrolled in the Stage I Science Communication course. I taught statistics to this class for five weeks. The criteria for selection were that the students were willing to participate and seemed to offer me an opportunity to gain an insight into some students' thinking. The students had not been taught by me when they volunteered to be interviewed.

Table 4.2 Second Exploratory Study Subject Data

Subject	Gender	Age Group	School Maths Level	Statistics Level	Personal Background
1	F	17-19	F7	Enrolled Stage I	Student
2	F	17-19	F7	Enrolled Stage I	Student
3	M	40+	F7	Stage II	Professional
4	M	20-24	F5	Enrolled Stage I	Semi-skilled
5	M	40+	F5	Enrolled Stage I	Unskilled
6	M	30-39	F5	None (used statistics in job)	Technician

Phase One: First Interview

Four weeks before the course started six students were interviewed. At the time of the interview the students who were enrolled in Stage I statistics had completed the same course work as the students in Exploratory Study One. The students were interviewed

individually for about one hour. They were told that I was interested in their thinking and reasoning rather than in their getting the 'correct' answer. Consequently they were asked to think aloud during the interview. The questions were presented orally and on paper. Unplanned probes were used in order to clarify the students' thinking for me. The interviews were audio-taped. Some items from Exploratory Study One that were regarded as appropriate for revealing students thinking were used, as well as more statistically based media-type articles (see Appendix Two).

Phase Two: The Five-Week Course

At the beginning of the course all students were given a questionnaire with four open-ended statistically based items to answer. The course focussed on teaching probabilistic and deterministic ideas in the context of statistics, with an emphasis on experiencing and explaining random and systematic variation. A key issue was developing students' awareness of their own thinking. The course was taught in two blocks, two weeks and three weeks and spanned a ten week period. Data exploration and simulations were a facet of the computer component of the course (5 hours), using *R* (Gentleman & Ihaka, 1994) and *DataScope* (Konold & Miller, 1995) software. For the lecture component (10 hours) experiments, media articles from newspapers and tables of data with background information (Finlay & Lowe, 1993) were used to enhance students' posing of questions and critical attitude. At the end of the course all students were given a questionnaire with open-ended and multi-choice items.

Phase Three: The Follow-Up Interview

Five weeks after the course a second interview with five of the students took place to further investigate their statistical thinking, beliefs and intuitions. The procedure was similar to the first interview. The students were each interviewed for one hour. The questions were presented orally and/or on paper depending on the nature of the question and one question involved exploring a data set on the computer using *DataScope* (Konold & Miller, 1995), which the students had used in the course (see Appendix Two). The students who were enrolled in Stage I statistics had, by this stage, completed most of the course including significance testing. Again the students were asked to 'think aloud' and unplanned probes were used.

Decision

A decision was made at this stage to refocus the study on the characteristics of statistical thinking, not on whether student thinking could be further developed through a different approach to teaching. The reasons were: (1) a teaching intervention was inappropriate in a large class (> 60) in a university environment as students did not attend every class and therefore the data were unreliable; (2) the area of study was too large; (3) there was an

awareness that the nature of the thinking skills that were being attempted to be addressed were not well understood either by me or my supervisor or by the profession at large; and (4) the data and issues that were being raised suggested that this avenue should be explored. Thus the decision was that the research should be focussed on understanding the nature of statistical thinking. With this refocussing, the data from the interviews of the first two exploratory studies, would be used for hypothesis generation about students' thinking, in relation to their reactions and solutions to given information, or to statistical investigations that had been carried out by someone else.

In order to capture and to explore more ideas about statistical thinking we made the design decision to carry out two more exploratory studies from two different perspectives: statisticians; and students involved in statistical investigations. The stated aims of the studies were:

1. To further explore the characteristics of statistical thinking.
2. To corroborate and expand on conjectures about statistical thinking stemming from the previous two exploratory studies.

4.5.2.2 Exploratory Study Three: The Statistician's Perspective

This exploratory study was started at the end of 1996. After interviewing some undergraduate students in two separate exploratory studies (Pfannkuch & Brown, 1996; Pfannkuch, 1996) on their solutions and reactions to statistically based information and problems, some hypotheses were formed on the nature of their statistical thinking. In order to broaden and understand the findings from these studies we decided to investigate the nature of statistical thinking from a practitioner's or 'expert's' perspective.

Subjects

The statisticians (Table 4.3) selected for the study were in the position of commenting, either from reflection on their own statistical thinking, and/or on their clients' or students' statistical thinking. Through this interaction with 'experts' it was hoped that further insights would be gained into the characteristics of statistical thinking and that this would be informative for conjecturing the type of statistical thinking that should be developed in students. It would also confirm or refute previous findings which had been solely derived from student data.

The criteria for selection were that the statisticians were: (1) willing to participate; (2) currently working as statistical consultants; and (3) seemed to offer the opportunity for me to learn. Most had also observed advanced students and clients grappling with real

statistical investigations and so were able to draw on students' and clients' difficulties as well as their own experience.

Table 4.3 Third Exploratory Study Subject Data

Subject	Gender	Age Group	Student Project Supervisor	Background	Main Field of Work
1	M	40+	Yes	Statistician	Science
2	M	40+	Yes	Statistician	Quality Assurance
3	M	40+	Yes	Biologist	Biological Science
4	M	40+	Yes	Statistician	Brain Mapping
5	F	25-29	No	Statistician	Medical
6	F	25-29	No	Statistician	Market Research

The Interview

Six statisticians were individually interviewed in depth. Broad questions were asked regarding their perceptions of the nature of statistical thinking and then the interview followed a semi-structured protocol based on the statistical enquiry empirical cycle (see Appendix Three). The interviews were approximately one and half hours long and could be regarded as a peer interaction that explored ideas and conjectures about statistical thinking and statistical investigations. If the conversation appeared to be leading down a particularly interesting track, then that avenue was pursued. The questions were presented orally and were audio-taped. Unplanned probes were used in order to clarify the perspectives of the subjects. The transcripts of the audio-tapes together with the analysis and interpretation of their comments were presented to and discussed with them for corroboration.

4.5.2.3 Exploratory Study Four: The Student-as-Investigator Perspective

This study was started at the end of 1996 and continued through to the end of 1997. It investigated the nature of statistical thinking from a student perspective.

Subjects

The students (Table 4.4) selected for the study were in the position of commenting, based on reflection of their own statistical thinking and actions, on the process of their statistical investigation. The criteria for selection were that the students were: (1) willing to

participate; (2) currently undertaking a statistical project of their own; and (3) seemed to offer the opportunity for me to learn. None of these students were known to me.

Table 4.4 Fourth Exploratory Study Subject Data

Subject	Gender	Age Group	School Maths Level	Statistics Level	Personal Background
1	M	17-19	F7	Enrolled Stage I	Student
2	F	20-24	F7	Enrolled in 6 Stage III	Student
3	F	20-24	F7	Enrolled in 3 Stage III	Student
4	F	20-24	F7	Enrolled in 1 Stage III	Part-time student
5	F	20-24	F7	Enrolled in 3 Stage III	Student

The Interview

Before conducting the interviews, I listened to the subjects' presentation of their projects in a seminar. The students were individually interviewed in depth. Broad questions, following a semi-structured protocol based on the statistical enquiry empirical cycle, were asked (see Appendix Four). The interviews were approximately one hour long. The questions were presented orally and were audio-taped. Unplanned probes were used in order to clarify the perspectives of the subjects. The transcript of the interview and the analysis and interpretation of her comments was able to be presented to and discussed with Subject 2 for corroboration. The other subjects were given their transcripts and the analysis and interpretation of their comments. They were given the opportunity to confirm or refute the interpretation of their data.

[Note: All the subjects who participated in the four exploratory studies were Caucasian.]

4.5.3 Design Decisions made at the End of the Research

We decided that final data analysis could begin. As a way of drawing common themes from the statisticians' interviews, categories were proposed and verified using *NUD•IST* (Richards & Richards, 1995) qualitative data analysis software (Richards & Richards, 1994). These categories were further developed and tested in an analysis of the project-student interviews, and then further tested on some first-and-second-exploratory-study student interviews. In order to ascertain the viability of the framework developed for statistical thinking (see Chapter 9), judgement criteria for the interpretation of statistically based information were constructed. Using these criteria, solution rubrics were created for some items in the second exploratory study. These rubrics were then used to judge the students' responses.