Chapter One
Introduction

1.1 Introduction and Background

Learning statistics requires not only obtaining a knowledge base but also obtaining particular ways of thinking. The knowledge base is overt and well documented whereas the particular ways of thinking are implicit and couched in vague generalities. Thus my thesis examines and explores the characteristics of the statistical way of thinking in an attempt to make what is largely intuitive more explicit. The journey into this territory is grounded in sixteen years of teaching experience in a large multicultural co-educational New Zealand secondary school. In my eight years as Head of the Mathematics Department there was a continual questioning of the teaching methods we were employing in the classroom and of the mathematics content we were teaching. One particular incident in 1986, that could be regarded as a pivotal moment in my teaching experience, led me to reflect on the reasoning processes of my students.

The National Bursary examination for the seventh form Mathematics With Statistics programme had an internally assessed project component. Before the students embarked on their own individual project I considered it important that they should experience the process of an investigation in the first term. About forty students were taken to Huia Beach to investigate the state of the cockle population. Ten quadrats were evenly spaced out from the low tide mark to the high tide mark and each group of students was required to count, measure and weigh every cockle found in the quadrat. A map of the beach was given to every student on which they drew the placement of the quadrats and features of the beach such as the pathways of two fresh water streams flowing onto the beach. The students collated all the data and dutifully drew box and whisker plots of the widths of the cockles in each quadrat. The median trend was bell-shaped with an anomaly in the sixth quadrat which plummeted down. The interpretation of the graph by the students was a revelation. There was no connection made between the beach and their data. Not one student related the sixth quadrat anomaly to the fact that the fresh water stream ran through it. Nor did they link the findings of fewer cockles at the low tide mark with the presence of other shellfish. Somehow previous school statistics projects and class work had failed to develop the notion that reasoning in statistics required the integration of the data with the context. Questions arose as to why our teaching approaches had not worked and what changes we should make to our teaching.

These vague ideas and uncertainties about developing students' statistical thinking were reawakened in 1988 with a new position in education which gave me the opportunity to finally read some research. In particular the writings of Moore (1990) on the core elements of statistical thinking started a research journey that was further fuelled when I met a statistician who wrote the following about our 1995 encounter:

> This research had its genesis in a clash of cultures. Chris Wild is a statistician. Like many other statisticians, he has made impassioned pleas for a wider view of statistics in which students learn "to think statistically" (Wild, 1994). Maxine Pfannkuch is a mathematics educator whose primary research interests are now in statistics education. Conception occurred when Maxine asked "What ***is*** statistical thinking?" It is not a question a statistician would ask. Statistical thinking is the touchstone at the core of the statistician's art. But, after a few vague generalities, Chris was reduced to stuttering.

1.2 The Need for the Study

The increasing use of technology to perform the mechanical operations and procedures of statistics has led to a questioning of what and how we should teach. As part of this rapid change new ways of conceiving statistics are being advocated (Chambers, 1993; Moore, 1997) with an increasing focus on defining (Cobb & Moore, 1997), and developing the statistical thinking skills of students (Bailar, 1988; Cobb, 1991; Snee, 1993; Wild, 1994). There is some research on how students think in a data-based environment, but little research on the type of thinking that is required for statistics. Apart from statisticians and statistics educators (Biehler, 1994b) reflecting on their own experience, there has been no research undertaken to develop a framework or theory specifically for statistical thinking in a data-based environment, that could be tested against reasoning processes. In fact, Mallows (1998) based his 1997 Fisher Memorial lecture around the need for the development of a theory for understanding the thinking processes used in applied statistics. He specifically mentioned that this theory should be useful for instruction. According to Resnick (1987) it is important for each discipline to set the criteria *"for what constitutes good reasoning and thinking"* (p. 48) and to embed its particular characteristic ways of reasoning into its teaching, as there is no evidence that general thinking skills are transferred. Furthermore it is crucial to practise reasoning with the discipline knowledge base as *"one must reason about something"* (p. 35). Thus the particular ways of reasoning in statistics ought to be grounded in data derived from the actual practice of statistics. For the theory to be explicit to the discipline of statistics it must reveal how the thinking skills are specifically used with that knowledge base.

Apart from the statisticians in Quality Management (e.g. Snee, 1990; Joiner & Gaudard, 1990), who have developed tools for thinking statistically, based on their intuition and experience, there has been no framework developed from an analysis of the reasoning processes used by statisticians and statistics students. I believe that we have not embedded the development of statistical thinking into the teaching process because we have not analysed how we think in our discipline. Until we analyse how we use general and specific thinking skills in statistics we will not teach them nor develop thinking tools or strategies that will help students reason statistically. This study provides a first attempt at such an analysis which can be subsequently revised, modified, extended and changed with further analyses of statisticians' and statistics students' reasoning.

1.3 My Research Question

What are some characteristics of statistical thinking, at the macro-level, in the domain of the empirical enquiry cycle from problem formulation to conclusions?

The purpose of this research is to inform teaching practice. The scope of the research will be entirely within the discipline itself and the data will be interpreted from the perspectives of a teacher-practitioner and a statistician. Thus my focus will be on developing a framework that characterises some aspects of statistical thinking that will be informative and be readily interpretable for and applicable to teaching. The framework should act as a springboard for developing different approaches and foci for teaching and act as a challenge to current teaching practices.

1.4 Overview of Chapters

This Chapter provides an introduction to the research.

Chapter Two gives an historical overview of the emergence of statistical thinking and refers to the debate on the links between statistics, probability and mathematics.

The literature review in Chapter Three, therefore, examines applicable research in the three content areas of mathematics, probability and statistics. The review discusses research in mathematical problem solving; in psychology; in thinking in a data-based environment from the perspective of statisticians and statistics educators; and discusses

some current theoretical models for stochastic conceptual development that were perceived as relevant to the research question.

Chapter Four reports on the nature and design of my research. It defines the research as qualitative and exploratory and gives a detailed description of the approaches used in each of the four exploratory studies. The criteria used for subject selection and a description of these subjects is included. Reference is made to research design assumptions, researcher biases and the methods used to overcome research design biases. A three stage framework for the design is used.

Chapters Five, Six and Seven report on the results, ongoing analyses, and interpretation of the data with reference to the literature, for each of the first three exploratory studies respectively.

Chapter Eight illustrates how part of the framework for characterising statistical thinking, which is presented fully in Chapter Nine, is derived from all four exploratory studies. It also demonstrates how that framework explains the statistical reasoning processes used by the subjects.

In Chapter Nine the complete four-dimensional framework for some characteristics of statistical thinking in the broad problem solving domain is presented. There is also a discussion on variation, causation and the development of thinking tools with reference to the literature.

The framework from Chapter Nine is used in Chapter Ten to develop some criteria for judging students' reactions to statistically based information. Armed with these criteria, a re-analysis of some data from exploratory study two is undertaken. Some conclusions are formed.

Chapter Eleven discusses the framework in terms of the literature, gives general conclusions and the implications of the research.