help them recognize the misinterpretation or mis-generalization" (*ibid,* p. 7). From anecdotal evidence, the authors noted that these activities have been largely successful in accomplishing this task.

More recently, different research teams have been spearheading innovations in the teaching and learning of linear algebra. Cooley, Martin, Vidakovic, and Loch (2007) developed a linear algebra course that combines the teaching of linear algebra with learning about APOS theory. By focusing on a theory for how mathematical knowledge is generated, students were made aware of their own thought processes and could then enrich their understanding of linear algebra accordingly. Other researchers have been working with Models and Modeling (Lesh & Doerr, 2003) and APOS to develop instruction that leverages students' intuitive ways of thinking to teach linear algebra. For example, Possani, Trigueros, Preciado, and Lozano (2010) utilized a genetic composition of linear independence and dependence and systems of equations in order to aid in the creation of a task sequence. The task sequence, which asked students to model the coordination of the traffic flow in a particular area of town, was designed to present students with a problem that they could first mathematise and then use to understand linear independence and dependence.

In the United States, another group of researchers has been drawing on sociocultural theories (Cobb & Bauersfeld, 1995) and the instructional design theory of Realistic Mathematics Education (Freudenthal, 1973) to explore the prospects and possibilities for improving the teaching and learning of linear algebra. Using a design research approach (Kelly, Lesh, & Baek, 2008), these researchers are simultaneously creating instructional sequences and examining students' reasoning about key concepts such as eigenvectors and eigenvalues, linear independence, linear dependence, span, and linear transformation (Henderson, Rasmussen, Zandieh, Wawro, & Sweeney, 2010; Larson, Zandieh, & Rasmussen, 2008; Sweeney, 2011). For example, these authors examined students' various interpretations of the equation A [x y] = 2 [x y], where [x y] is a vector and A is a 2 x 2 matrix prior to any instruction on eigentheory. They identified three main categories of student interpretation and argue knowledge of student thinking prior to formal instruction is essential for developing thoughtful teaching that builds on and extends student thinking. This group has also begun to disseminate studies on the sequences of tasks for developing student reasoning of basis and constructing understanding of vectors, vector equations, linear dependence and independence and span. For example, Wawro, Zandieh, Sweeney, Larson, and Rasmussen (2011) report on student reasoning as they reinvented the concepts of span and linear independence. The reinvention of these concepts was guided by an innovative instructional sequence that began with vector equations (versus systems of equations, that most introductory texts employ) and successfully leveraged students' intuitive imagery of vectors as movement to develop formal definitions. This more recent work challenges some of the earlier findings that students' intuitive ways of reasoning are an obstacle to induction into formal mathematics.

Drawing on the TDS and ATD frameworks de Vleeschouwer and Gueudet (2011) put forward the view that some of the difficulties students experience may originate in the institutional experiences they have been offered (e.g. tasks). However, they observe that students can learn to appreciate the duality in linear forms (process-object or, to these authors, micro-macro) if given an appropriate set of tasks that require them to engage with these concepts at both levels. Their perspective is that of the changing didactical contract between school and university mathematics, particularly with regard to ways of approaching mathematical content (and less of the more common research foci on more general aspects of the students' mathematical learning experiences such as teacher expectations, attitudes to proof etc.).

Discrete Mathematics

Discrete mathematics deals with finite or countable sets, bringing into play several overlapping domains, e.g. number theory, graph theory, and combinatorial geometry (Grenier, 2011). It occupies a rather variable place in mathematics education; in some countries, only a very small number of discrete mathematics concepts are taught, often those related to combinatorics and the basics of number theory. Discrete mathematics can be introduced, either as a mathematical theory, or as a set of tools to solve problems (a graph is a basic and intrinsic modelling tool). For example, mathematical games are often based on problems in discrete mathematics. We present here three contributions that illustrate how discrete mathematics can be used in mathematics in both high school and university for addressing important issues in the transition such as the nature and elaboration of mathematical definitions, and reasoning modes such as for instance reasoning by induction, necessary and sufficient conditions. Using contexts such as the Königsberg's bridges problem is suggested by Cartier and Moncel (2011) as a way to provide access to fundamental mathematical concepts like proofs, necessary and sufficient conditions, and modelling techniques. The elaboration of definitions was the topic of Ouvrier-Buffet (2011). She suggests that encouraging students to work on skills such as defining, proving and conceiving new concepts through various discrete mathematics concepts, such as trees, discrete straight lines and properties of displacements on a regular grid to generate knowledge and concepts. These types of activity give rise to specific reasoning modes and the potential for construction of new tools, such as coloring, proof by exhaustion of cases, proof by induction, and use of the Pigeonhole principle (Grenier, 2001, 2003). In this manner misconceptions that persist in the knowledge of many students as they are in transition to university, such as inaccurate knowledge about mathematical induction, may be addressed.

Logic and Proof

The difficulties met by transition students concerning logic are well recognized by teachers and mathematics educators around the world. In France, research on the role of logic in the learning and teaching of mathematics, and more specially proof and proving, has been developed since the eighties. Durand-Guerrier (2003), as well as Deloustal-Jorrand (2004) and Rogalski and Rogalski (2004) point out the importance of taking in account quantification matters in order to analyse difficulties related to implication, and more generally mathematical reasoning. In the same vein, in a Tunisian context, Chellougui (2004, 2009) investigates the use of quantification by new university students in Tunisia. Her didactic analysis of textbooks and course notes concerning upper limit, as well as an interview with pairs of students in a problem-solving situation, revealed, on the one hand, the didactic phenomena related to the alternation of the two types of quantifiers and, on the other hand, difficulties in mobilizing the definition of the objects and the structures, which illustrate a major problem in the conceptualization process. These authors, as well as Durand-Guerrier and Arsac (2003, 2005) acknowledge that the importance of these questions seems to be largely underestimated by teachers at secondary school as well as at university level, as it appears in particular in textbooks. Durand-Guerrier and Arsac (2003, 2005) highlight the fact that a main challenge for novices is to develop together mathematical knowledge and logical skills, which are closely intertwined. Durand-Guerrier (2005) supports the relevance of a model theoretic point of view for analysing proof and proving in mathematics. These pieces of research concern mostly written mathematical discourse. In order to study deeply the oral interaction in argumentation and proof, Barrier (2009a, 2009b) introduces a semantic and dialogic perspective as developed by Hintikka. This permits one to highlight the importance of moving back and forth between syntax and semantics in the proving process in advanced mathematics (e.g. Blossier, Barrier & Durand-Guerrier, 2009). This research, together with research in other areas, calls for developing programmes allowing new university students to master the logical competencies required for the learning of advanced mathematics. The role of acquiring these competencies in a way that is similar to second language learning is developed by Durand-Guerrier and Njomgang Ngansop (2011), in a continuation of the work of Ben Kilani (2005) at secondary level.

A previous ICME survey report on proof (Mariotti et al., 2004) raised a number of questions that relate to transition issues. Among these were: “Is proof so crucial in the mathematics culture that it is worthwhile to include it in school curricula?”; “What are the meanings of proof and proving in school mathematics and how are these meanings introduced into curricula in different countries?”. Important aspects include: students’ conceptions of proof, students’ performance in proof tasks; teachers’ conceptions of proof; and, how research in mathematics education has approached the issue of proof. Of particular interest has been the question “is it possible to overcome the difficulties in introducing pupils to proof so often described by teachers?” (Mariotti et al., 2004, p. 184).

The key difference between school and university, which is expressed as a possible rupture, is that schools focus on argumentation while universities consider deductive proof (Mariotti et al., 2004, p. 193). Iannone and Inglis (2011) discuss a range of weaknesses in newly arriving Year 1 mathematics students’ production of deductive arguments (rather than in the oft-reported perception that a deductive argument was expected of them). Specifically, Year 1 mathematics students responded to four proof tasks and while they demonstrated a range of weaknesses in their production of deductive arguments, they were aware that when asked to generate a proof, they should provide a deductive argument. This is in some contrast to previous work in the field but this contrast may be accounted for by different student background and specialisms in the student sample.

In a translation of his own paper (Balacheff, 1999), Balacheff argues for the notion of *Cognitive Unity* (Boero, Garuti & Mariotti, 1996) as a potential bridge between them, saying “I would summarize in a formula the place that I find possible for argumentation in mathematics, according to the notion of Cognitive Unity as it was introduced by our Italian colleagues: argumentation relates to conjecture, like proof does to a theorem” (Mariotti et al., 2004, p. 194). The survey report further proposed that "Research studies concerning the analysis of argumentation processes and their comparison with the production of mathematical proof appear to be very promising" (Mariotti et al., 2004, p. 201). The suggestion by Heinze and Reiss (2003) was that schools should move students away from inductive arguments toward formal argumentation. More recently, Antonini and Mariotti (2008) have applied the Cognitive Unity framework to the application of indirect proofs, such as contradiction and contraposition. They suggest that this provides a perspective, taking into account both epistemological and cognitive considerations, from which one may observe the relationship between argumentation and proof by focusing on analogies, without forgetting differences.

The 2004 survey report further recommended a cautious approach, suggesting that the inclusion of proof in the school or university curriculum is only a first step, and it is important to ensure that the goals for doing so should be clarified, along with processes for how they will be operationalized (Mariotti et al., 2004). In the years since that report there have been many studies considering the role of proof, both at school and university. However, there appear to have been few studies directly addressing proof as an issue of transition (we note that at the time of writing the book *Proof and Proving in Mathematics Education*: *The 19th ICMI study* (Hanna & de Villiers, 2012) was still in print). While this is the case, the research does point out some of the key differences between approaches to proof in school and in university and makes suggestions for pedagogical approaches that might assist in the transition. In this section we draw on some of these aspects of proof studies.

One theoretical perspective that may prove useful in considering the role of proof in transition is that of Harel (2008a, b), who proposes a framework called DNR-based instruction, which involves duality (D), necessity (N) and repeated reasoning (R). In this he distinguishes between ways of understanding, a generalisation of the idea of proof, and ways of thinking, which generalises the notion of proof scheme, but also includes problem solving approaches and beliefs about mathematics. In general, proof schemes are present at school, while learning and understanding in university is via proofs. One of the principal implications of defining mathematics as comprising both aspects is "that mathematics curricula at all grade levels, including curricula for teachers, should be thought of in terms of the constituent elements of mathematics—ways of understanding and ways of thinking—not only in terms of the former, as currently is largely the case." (Harel, 2008, p. 490). However, such a definition of mathematics is consistent with mathematicians' practice of mathematics, but not with their perception of it. There is a fundamental difference between the way mathematicians perceive mathematics and the way they practice it in their research. One reason for this may be, as Hanna and Janke (1993 – cited in Balacheff, 2008) hypothesise, that "Communication in scholarly mathematics serves mainly to cope with mathematical complexity, while communication at schools serves more to cope with epistemological complexity." (Balacheff, 2008, p. 433).

According to Solomon (2006), enabling students to access academic proof processes in the transition from pre-university to undergraduate mathematics is a question of understanding and building on students' own pre-existing epistemological resources in order to foster an epistemic fluency that will allow them to recognize, and engage in, the process of creating and validating mathematical knowledge. Since transition involves maturation and its accompanying changes in thinking, Tall and Mejia-Ramos (2006) apply TWM to outline the changes in proof types that they suggest occur as students become more mathematically sophisticated. Firstly, in the embodied world, the individual begins with physical experiments to find how things fit together. Then in the symbolic world, arguments begin with specific numerical calculations and develop into the proof of algebraic identities by symbolic manipulation. However, it is only in the formal world where proof by formal deduction occurs. Thus as students develop cognitively, moving through the three worlds, their argument warrants (Toulmin, 1958) change, and the hope is that formal proof will become the only acceptable warrant. Tall (2004) refers to this as moving through the 'three worlds' of mathematics and characterizes development through the worlds, which impinges on production of proof schemes, as a move from perception and action, through operation and symbolism, to reason and formality (Tall, Yevdokimov, Koichu, Whiteley, Kondratieva & Cheng, 2012). Pinto and Tall (2002) also describe *natural thinking* as using thought experiments based on embodiment and symbolism to give meaning to definitions and to suggest possible theorems for formal proof.

Among the recommendations for pedagogical change that would have implications for transition is the point made by Balacheff (2008) and others (eg Hanna & de Villiers, 2008; Hemmi, 2008) that there is a need for more explicit teaching of proof, both in school and university. Some, (e.g., Stylianides & Stylianides, 2007; Hanna & Barbeau, 2008) argue for it to be made a central topic in both institutions. One reason given by Hanna and Barbeau (2008) is that, apart from their intrinsic value, proofs may display fresh methods, tools, strategies and concepts that are of wider applicability in mathematics and open up new mathematical directions for students. One example they cite, applicable to transition, is that an algebraic proof of the formula for solving a quadratic equation introduces the technique of adding a term and then subtracting it again. Hence they argue that "…proofs could be accorded a major role in the secondary-school classroom precisely because of their potential to convey to students important elements of mathematical elements such as strategies and methods." (Hanna & Barbeau, 2008, p. 352). One way to make proof more central in the school mathematics classroom, proposed by Heinze et al. (2008) is the use of heuristic worked-out examples as an instrument for learning proof. While these kind of examples are based on traditional worked-out examples, they make explicit the heuristics of the problem solving process. The research showed some success with low- and average-achieving students, but there was no significant effect for high-achieving students (*ibid*). However, if proof is made more central, Balacheff, (2008) cautions that teaching of mathematical proof "must not lead to an emphasis on the form, but on the meaning of proof *within* the mathematical activity." (p. 506). Further, he maintains that to understand what proving is about requires the systematic organization of validation (eg control), communication (eg representation) and the nature of knowing. Three requirements for successful engagement with proof are also listed by Stylianides and Stylianides (2007): to recognize the need for a proof; to understand the role of definitions in the development of a proof; and the ability to use deductive reasoning.

Two potential difficulties in any attempt to place proving more prominently in the transition years are the role of definitions, and the problem of student met-befores (Tall & Mejia-Ramos, 2006). A desire to use definitions as the basis of deductive reasoning in schools is likely to meet serious problems, since, according to Harel (2008), this form of reasoning is generally not available to school students. In fact he claims "…it does not become an integral part of the repertoire of students' ways of thinking until advanced grades (if at all)… Understanding the notion of mathematical definition and appreciating the role and value of mathematical definitions in proving is a developmental process, which is not achieved for most students until adulthood." (Harel, 2008, p. 495).

Evidence for this is that when asked to define an invertible matrix, many linear algebra students stated a series of equivalent properties (e.g., "a square matrix with a nonzero determinant", ''a square matrix with full rank", etc.) rather than a definition. The conclusion is that the provision of more than one such property indicates that they were not thinking in terms of a mathematical definition (Harel, 2008). A study by Hemmi (2008) agrees that students have difficulties understanding the role of definitions in proofs and lack experience of proving in their secondary school mathematics. She advocates a style of teaching that uses the principle of *transparency*, making the difference between empirical evidence and deductive argument visible to students. In this manner proof techniques, key ideas, structures of proofs could be taught at the same time as proof is used by the teacher and the students to verify convince and explain mathematics. Her study showed that for students many aspects of proof remained invisible and they often wondered exactly what constituted a proof, since there were no discussions about proof or proof techniques for students new to it. Adding transparency would avoid students being left to find out by themselves and judge if their solutions are correct, and why. A study by Cartiglia et al. (2004) showed that the cognitive influence of student met-befores (Tall & Mejia-Ramos, 2006) was strong, with the most recent met-before for university students, namely a formal approach, having a strong influence on their reasoning. Having formed the habit of using formal mathematical knowledge as the only resource for doing mathematics inhibited their ability to look for meaning in algebraic formulas.

Another possible difficulty is the form of teaching in schools. It has been suggested that one of the major differences between argumentation and mathematical proof that could lead teachers to advance mostly argumentation skills with little or no deductive reasoning is the need to distinguish between the status and content of a proposition (Duval, 2002; Harel, 2008). A potential way forward, proposed by Inglis, Mejia-Ramos, and Simpson (2007), is the use of the full Toulmin argumentation scheme, including its modal qualifier and rebuttal. Their research indicates "non-deductive warrant-types play a crucial role in mathematical argumentation, *as long as they are paired with appropriate modal qualifiers*… they retain the use of the warrants that have been used in previous 'worlds' or 'proof schemes,' but *they qualify them appropriately* (where appropriateness is defined by expert practice)." (Inglis, Mejia-Ramos, & Simpson, 2007, p. 17) This has possible implications for transition, since it would not be necessary for teaching to go straight to the use of formal deductive warrants.

A positive pedagogical approach to the teaching of proving proposed by a number of researchers (eg Kondratieva, 2010; Pedemonte, 2007, 2008) is student construction and justification of conjectures. Pedemonte's (2007) conclusion was that teaching of proof based on presentation of proofs to students and getting them to reproduce them, rather than to construct them, appears to be unsuccessful. Instead she highlights the need for open problems that ask for a conjecture, which appears to be a very effective way to introduce the learning of proof. She also discusses (Pedemonte, 2007, 2008) the relationship between argumentation and proof in terms of *structural distance*, moving from abductive, or plausible, argumentation to a deductive proof, where in the former inferences are based on content rather than on a deductive scheme. She argues for an abductive step in the structurant argumentation (coming after a conjecture, to justify it), since it "could be useful in maintaining the connection between the referential system in the constructive argumentation [contributing to construction of a conjecture] and the referential system in the proof, because it could help students to maintain the meaning of numerical examples used to construct the conjecture and algebraic letters used in the proof." (Pedemonte, 2008, p. 390). In this way it is hoped that the abductive step would decrease the gap between the arithmetic field in argumentation and the algebraic field in proof, and thus assist transition.

Another pedagogical approach, presented by Kondratieva (2010), uses the idea of an interconnecting problem to get students to construct and justify conjectures. The problem should allow simple formulation, solutions at various levels, be solvable using tools from different mathematical branches, and appropriate for different contexts. The value of conjecture production has also been espoused (Antonini & Mariotti, 2008) during production of indirect proofs, such as by contradiction and contraposition. The research, using a Cognitive Unity approach, showed that the production of indirect argumentation can hide some significant cognitive processes. Hence, they propose that task of producing a conjecture offers students the possibility both of activating these processes and of constructing a bridge to overcome the gaps. The conclusion is that "…without any conjecturing phase, some gaps could not be bridged or could require sacrifices and mental efforts that not all the students seem to be able to make." (Antonini & Mariotti, 2008, p. 411).

Two possible strategies to prepare upper secondary school students for transition to the rigour of tertiary proofs suggested by Yevdokimov (2003) include: the value of intuitive guesses, and experience in what distinguishes a reasonable guess from one that is less reasonable; and a consideration of restrictions on statements and proofs. This idea of considering restrictions, which links to ideas about the status of a proposition (Duval, 2002; Harel, 2008), has led some to propose the idea of pivotal and bridging examples, and suggest that a strategy using counterexamples can assist students with proof ideas (Zazkis & Chernoff, 2008). These authors claim that one benefit of a counterexample is to produce cognitive conflict in the student, and a pivotal example is designed to create a turning point in the learner's cognitive perception (*ibid*). In a similar vein Stylianides and Stylianides (2007) state that counterexamples also foster deductive reasoning, since we make deductions by building models and looking for counterexamples. For Zazkis and Chernoff (2008) a counterexample is a mathematical concept, while a pivotal example is a pedagogical concept, and it is important that pivotal examples are within, but pushing, the boundaries of the student's *potential example space* (Watson & Mason, 2005 – the examples students have experienced). The importance of developing mathematical thinking through extension of example spaces by the addition of examples and counterexamples has been advocated by Mason and Klymchuk (2009). Another way to expand students' example spaces, researched by Iannone et al. (2011), was based on Dahlberg and Housman's (1997) idea that getting students to generate their own examples of mathematical concepts might improve their ability to produce proofs. However, the results did not support the hypothesis that generating examples is a more effective preparation for proof production tasks than reading worked examples. These authors conclude that this may be because of the examples employed, and believe that there is currently insufficient guidance available on how to generate suitable examples effectively (Iannone et al., 2011). The role of examples also arose in research by Weber and Mejia-Ramos (2011) on how to read proofs. They looked at proof reading by mathematicians and found that they were mainly concerned with understanding the key ideas, the structure and the techniques employed. Hence they suggest that "One implication for the design of learning environments is that students might be taught how to use examples to increase their conviction in, or understanding of, a proof in the same way that the mathematicians in this paper described the ways that they read proofs." (Weber and Mejia-Ramos, 2011. p. 14). One of these ways is that they might see the value or insight that understanding a proof may provide for them personally.

A pedagogical strategy propose by Yevdokimov (2003) is that a way to arouse interest and free students from the monotony of 'standard' problems is to give them questions such as to find the mistakes in a given proof. However, when students check for errors in proofs they should be directed to consider three aspects of the methodological knowledge, proof scheme, proof structure and chain of conclusions (Heinze & Reiss, 2003).

Regardless of the route taken, there has been a discussion (Alcock & Inglis, 2008, 2009; Weber, 2009) on the relative roles of syntactic and semantic reasoning in proof construction. However, this seems to hinge on the definition of a syntactic proof, whether all, or just most, of the reasoning occurs within the representation system of proof. Alcock and Inglis (2008, 2009) argue that there are different strategies of proof construction among experts, and hence we need to identify these in order to know what skills we need to teach students and how they can be employed. They propose a need for large-scale studies to investigate undergraduate proof production, and an extension of this to include upper secondary school could be beneficial for transition.

One specific kind of problem that may be a good introduction to proof in schools, as suggested by Harel (2008), is one involving proof by mathematical induction. However, he claims that this method of proving is often considered too quickly and the DNR framework suggests that a slower approach is necessary for understanding (Harel, 2001). The research by Palla, Potari and Spyrou (2011) suggests that induction can be taught in a meaningful way at the upper secondary level if students are given tasks that encourage them to focus on the critical properties of mathematical induction. In addition, Man-Keung Siu (2008) recommends the use of history to help students engage with proof, thus humanising it, placing it in a cultural, socio-political and intellectual context. In a similar vein Nagafuchi (2009) presents some elements that would make a historical-philosophical approach possible for mathematical proofs in undergraduate courses and Furinghetti (2000) provided students with a historical presentation of 'definition' in an attempt to encourage flexibility, open-mindedness and motivation towards mathematics.

Our survey considered proof in several questions. In response to 'How important do you think definitions are in **first** year mathematics?' 52 (65.8%) replied that definitions are important in first year mathematics, while 15 presented their responses as neutral. Only 8 respondents replied that definitions are not important in first year mathematics. Responses to the question 'Do you have a course that explicitly teaches methods of proof construction?' were evenly split with 49.4% answering each of "Yes" and "No". Of those who responded "Yes", 15 (38.4%) replied that they teach methods of proof construction during the first year, 23 (58.9%) during the second year and 5 (12.8%) in either third or fourth year. While some had separate courses (e.g. proof method and logic course) for teaching methods of proofs, many departments teach methods of proofs traditionally, by introducing examples of proof and exercises in mathematics class. Some respondents replied that they teach methods of proof construction in interactive contexts, citing having the course taught as a seminar, with students constructing proofs, presenting them to the class, and discussing/critiquing them in small size class. One respondent used the modified Moore method in interactive lecture. Looking at some specific methods of introducing students to proof construction was the question 'How useful do you think that a course that includes assistance with the following would be for students?' Four possibilities were listed, with mean levels of agreement out of 5 (high) being: Learning how to read a proof, 3.7; Working on counterexamples, 3.8; Building conjectures, 3.7; Constructing definitions, 3.6. These responses appear to show a good level of agreement with employing the suggested approaches as components of a course on proof construction.

Mathematical Modelling and Applications

Blum et al. (2002) wrote in the Discussion Document of the 14th ICMI Study: "It is not at all surprising that applications and modelling have been – and still are – a central theme in mathematics education. Nearly all questions and problems in mathematics education, that is questions and problems concerning the learning and teaching of mathematics, affect, and are affected by, relations between mathematics and the real world." This might be the reason why research on mathematical modelling and applications has attracted an increasing interest in recent years. This trend can be noted from the fact that there is a growing research literature focusing on the teaching and learning of mathematical modelling and applications published in various mathematics education journals. In addition, there are also several international conferences/events dedicated to the teaching and learning of mathematical modelling and applications.

The literature reports many studies and practices on the teaching and learning of mathematical modelling and applications, for both the secondary and tertiary levels. The primary focus of much research is on practice activities, e.g. on constructing and trying out mathematical modelling examples for teaching and examinations, writing application-oriented textbooks, implementing applications and modelling into existing curricula or developing innovative, modelling oriented curricula (Blum et al., 2002). There are also extensive studies on clarifying modelling concepts, characterising the features of modelling processes, classifying the modelling tasks, and investigating what are and how to evaluate and improve the students' modelling competencies and sub-competencies required for each modelling process. However, it appears that no literature exists explicitly discussing this topic with a focus on the 'transition' from the secondary to the university levels. One reason might be that until now there are have been no roadmaps to sustained implementation of modelling education at all levels. As Blum et al. (2002) point out the role of applications and mathematical modelling in everyday teaching practice is still rather marginal for all levels of education. The big issue seems to be whether, and if so how, this trend can be reversed to ensure that applications and mathematical modelling is integrated and preserved at all levels of mathematics education.

There is recent literature partially relevant to the secondary-tertiary transition issue and this is briefly considered here. One crucial duality, mentioned by Niss et al. (2007), is the difference between 'applications and modelling for the learning of mathematics' and 'learning mathematics for applications and modelling'. They point out that in lower secondary levels this duality is seldom made explicit, and instead both orientations are simultaneously insisted on. However, at upper secondary or tertiary level the duality is often a significant one. Their analysis suggests that for students to develop applications and modelling competency as one outcome of their mathematical education, these have to be put explicitly on the agenda of the teaching and learning of mathematics.

The close relationship between modelling and problem solving is taken up by a number of authors and reports (see, for example, *Focus in High School Mathematics: Reasoning and Sense Making*, NCTM, 2009 and the *Common Core State Standards for Mathematics*, National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). For example, English and Sriraman (2010) suggest that mathematical modelling is a powerful option for advancing the development of problem-solving in the curriculum. However, according to Petocz et al. (2007), there are distinct advantages to using real world tasks in problem solving. They note that well-designed learning tasks that model the way mathematicians work can encourage students towards broader conceptions of mathematics, enabling explicit connections between students' courses and the world of professional work. One difficulty described by Ärlebäck and Frejd (2010) is that upper secondary students (in Sweden) do not have much experience working with real situations and modelling problems, making the incorporation of real problems from industry in the secondary mathematics classroom problematic. One possible solution they suggest is closer collaboration, with representatives from industry working directly with classroom teachers. A second potential difficulty arose in a survey of 62 secondary mathematics teachers by Gainsburg (2008): teachers don not tend to make many real-world connections in teaching. Reasons given for this were that it would take more time than teachers feel they can spend on most mathematics topics, it isn't stressed in the curriculum or assessment, and teachers feel a need for more resources, ideas, or training about what real world connections to make. One possible solution to this, suggested by the German experience, is to bring together combinations of students, teachers and mathematicians to work on modelling problems. An example of this approach is reported by Kaiser and Schwarz (2006) who describe their experience of modelling projects where prospective teachers together with upper secondary level students carry out modelling examples either in ordinary lessons or special afternoon groups. Further, Kaland et al. (2010) present experiences with modelling activities known as the “modelling week”, in which small groups of students from upper secondary level work intensely for one week on selected modelling problems, while their work is supported by pre-service-teachers. These activities are unique because they create a setting where pre-service teachers and upper secondary students are afforded the opportunity to work on authentic problems that applied mathematicians tackle in industry. In other studies on modelling activities, Heilio (2010) reports tertiary level experiences with a “modelling week” project for undergraduate students across Europe and Göttlich (2010) reflects experiences in conducting “modelling week” projects and modelling courses with students (especially secondary level and undergraduates) at the University of Kaiserslautern, describing how practical implementations can be performed. Another way to assist teachers proposed by Maaß (2010) is a scheme for modelling tasks that provides an overview of the different features of modelling tasks, thus offering guidance in task design and selection processes for specific aims and predefined objectives and target groups. According to Bracke (2010) his twenty years experience of modelling with students suggest that mathematical modelling should be integrated into teacher training, including the learning by doing component, training of the supervisor role and learning how to find problems. To achieve this he proposes including student teachers in organisation and implementation of modelling events in schools, as implemented at Technische Universität Kaiserslautern, with promising results.

Some difference between problem solving at school and university are identified by Perrenet and Taconis (2009), who investigated changes in mathematical problem-solving beliefs and behaviour of mathematics students during the years after entering university. They report significant shifts for the group as a whole, such as the growth of attention to metacognitive aspects in problem-solving or the growth of the belief that problem-solving is not only routine but has many productive aspects. The students explain these shifts mainly by the change in the specific nature of the mathematics problems encountered at university compared to secondary school mathematics problems, with the latter not succeeding in presenting an authentic image of the culture of mathematics with regard to problem-solving.

There is some agreement that there is a need to target curriculum changes in the upper secondary school to include more modelling activities. For example, in a summary discussion of perspectives on mathematical modelling and applications in upper secondary and tertiary levels, Stillman (2007) points out that high-stakes assessment at the upper secondary-tertiary interface is often seen as an unresolved problem for the infusion of modelling into the secondary curriculum at this level. Explaining that other imperatives are uppermost in the minds of teachers and students due to the pressure from the external examination system, she advocates authentic evaluation of current upper secondary assessment practices so future planning and policy can be based on actualities. Other possible initiatives in this direction were suggested by Stillman and Ng (2010), who recognised two different models of curriculum embedding intended to bring authentic real world applications into secondary school curricula. The first has a system-wide focus emphasising an applications and modelling approach to teaching and assessing all mathematics subjects in the last two years of pre-tertiary schooling. The second model involves interdisciplinary project work from upper primary through secondary school with mathematics as the anchor subject. Another initiative presented by Maaß and Mischo (2011) is the framework and methods of the project STRATUM (Strategies for Teaching Understanding in and through Modelling), whose aim is to design and evaluate teaching units for supporting the development of modelling competencies in low-achieving students at the German Hauptschule. Also, in the USA, Leavitt and Ahn (2010) have provided a teacher's guide to implementation strategies for Model Eliciting Activities (MEAs), which are becoming more popular in secondary schools. Another arena that might prove helpful to students making the secondary-tertiary transition in mathematical modelling and applications is entry to contests in mathematical modelling and applications, available to both high secondary and tertiary students. Examples include HiMCM (The High School version of the Mathematical Contest in Modeling) for high school students, MCM (Mathematical Contest in Modeling) and ICM (Interdisciplinary Contest in Modeling) for undergraduate students. each of these operated annually by the Consortium for Mathematics and it Applications (COMAP, see http://www.comap.com). In addition, there is CUMCM (Contemporary Undergraduate Mathematical Contest in Modelling) for undergraduate students (http://en.mcm.edu.cn). This international contest is operated annually by the Chinese Society for Industrial and Applied Mathematics (CSIAM) and each year there are more than 1,000 institutions and about 50,000 students participate in it (Xie, 2010).

Our survey addressed the topic of mathematical modelling in universities. In response to the questions "Does your university have a mathematical course/activity dedicated to mathematical modeling and applications?" Or "Are mathematical modelling and applications contents/activities integrated into other mathematical courses?", 44 replied that their departments offer dedicated courses for modelling, while 41 said they integrate teaching of modelling into mathematics courses such as calculus, differential equations, statistics, etc and 7 answered that their university does not offer mathematics courses for mathematical modelling and applications. Among the reasons given for choosing dedicated courses were that: the majority of all mathematics students will end up doing something other than mathematics so applications are far more important to them than are detailed theoretical developments; most of the mathematics teaching is service teaching for non-majoring students so it is appropriate to provide a course of modelling and applications that is relevant to the needs of the target audience; and if modelling is treated as an add-on then students do not learn the methods of mathematical modeling. Those who chose integrated courses did so because: for modeling, students need to be equipped with a wide array of mathematical techniques and solid knowledge base. Hence it is appropriate for earlier level mathematics courses to contain some theory, proofs, concepts and skills, as well as applications.

Considering what happens in upper secondary schools, 26 (33%) reported that secondary schools in their location have mathematical modelling and applications integrated into other mathematical courses, with only 4 having dedicated courses. 44 (56%) said that there were no such modelling courses in their area. When asked for their opinion on how modelling should be taught in schools, most of the answers stated that it should be integrated into other mathematical courses. The main reasons presented for this were: the many facets of mathematics; topics too specialised to form dedicated courses; to allow cross flow of ideas, avoid compartmentalization; and students need to see the connection between theory and practice, build meaning, appropriate knowledge. The question 'What do you see as the key differences between the teaching and learning of modelling and applications in secondary schools and university, if any?' was answered by 33 (42%) of respondents. The key differences pointed out by those answering this question were: at school, modelling is poor, too basic and mechanical, often close implementation of simple statistics tests; students have less understanding of application areas; university students are more independent; they have bigger range of mathematical tools, more techniques; they are concerned with rigour and proof. Asked 'What are the key difficulties for student transition from secondary school to university in the field of mathematical modelling and applications, if any?' the 35 (44%) university respondents cited: lack of knowledge (mathematical theory, others subjects such as physics, chemistry, biology, ecology); difficulties in formulating precise mathematical problems/interpreting word problems/understanding processes, representations, use of parameters; poor mathematical skills, lack of logical thinking; no experience from secondary schools; and lack of support.

BRIDGING THE GAP AND WAYS FORWARD

In order to address how universities respond to assist students with transition problems our survey asked "Do you have any academic support structures to assist students in the transition from school to university? (e.g., workshops, bridging courses, mentoring, etc).", and 56 (71%) replied 'Yes' and 22 'No'. Of those saying yes, 34% have a bridging course, 25% some form of tutoring arrangement, while 23% mentioned mentoring, with one describing it as a "Personal academic mentoring program throughout degree for all mathematics students" and another saying "We tried a mentoring system once, but there was almost no uptake by students." Other support structures mentioned included 'study skills courses', 'maths clinics', 'support workshops', 'pre-course', 'remedial mathematics unit', and a 'Mathematics Learning Service (centrally situated), consulting & assignment help room (School of Maths). The MLS has a drop-in help room, and runs a series of seminars on Maths skills. These are also available to students on the web.' Others talked of small group peer study, assisted study sessions, individual consultations, daily help sessions, orientation programmes and remedial courses.

There is some evidence that bridging courses can assist in transition. A recent study by (Tempelaar, Rienties, Giesbers & Schim van der Loeff, 2012) showed that an online summer course with a broad coverage of basic mathematical topics and learning controlled by individual, adaptive testing, was very efficient in addressing skill deficiencies, with the treatment effect of successful summer course participation about 50% of the effect size of advanced prior math education. A description by Carmichael and Taylor (2005) of a study of students in a supportive bridging mathematics course indicates that student confidence contributes significantly to performance, even after accounting for prior knowledge, and for some this may be because they struggle with their learning of mathematics in English at undergraduate level much more than is sometimes appreciated (Barton, Chan, King, Neville-Barton & Sneddon, 2005). In Australia, an increasing number of students elect not to undertake studies in mathematical methods in the final years of their secondary schooling, and hence some support structures are required. Some higher education providers offer pathways for these students to pursue mathematics studies up to a major specialization within the bachelor of science programme. The article by Varsavsky (2010) analyses the performance in, and engagement with, mathematics of students who elect to take up this option. Findings indicate that these are not very different when compared to students who enter university with an intermediate mathematics preparation. Leviatan (2008) presents details of a transition course aimed at bridging the gap for students of four-year secondary/high schoolteacher training programme. The objectives of this transition programme are: to identify and reinforce previous "core school mathematics"; to deepen and enrich the existing knowledge by adopting a more mature perspective to school mathematics; to introduce mathematical "culture" (language, rules of logic, etc.); to get acquaintance with typical mathematical activities (generalizations, deductions, definitions, proofs, etc.); to re-introduce central mathematical concepts and tools; and to provide a rigorous, yet only semiformal, exposure to selected new topics in advanced mathematics. Students' evaluations of the programme report increasing self-confidence, as well as enjoyment of the sessions about misconceptions and playing the role of a reviewer. She concludes that a more systematic investigation is required and suggests possible follow-up. In other cases, a university first year programme of tutor training and collaborative tutorials, reported by Oates, Paterson, Reilly and Statham (2005), proved an effective way of addressing some of the mathematical issues in the transition.

The literature review presented here revealed a multi-faceted web of cognitive, curricular and pedagogical issues, some spanning across mathematical topics and some intrinsic to certain topics – and certainly exhibiting variation across the institutional contexts of the many countries our survey focused on. For example, most of the research we reviewed discusses the students' limited cognitive preparedness for the requirements of university-level formal mathematical thinking (whether this concerns the abstraction, for example, within Abstract Algebra courses or the formalism of Analysis). Within other areas, such as discrete mathematics, much of the research we reviewed highlighted that students may arrive at university with little or no awareness of certain mathematical fields.

The literature review presented in this report is certainly not exhaustive. However we believe it is reasonable to claim that the bulk of research on transition is in a few areas (e.g. calculus, proof) and that there is little research in other areas (e.g. discrete mathematics). While this might simply reflect curricular emphases in the various countries that our survey focused on, it also indicates directions that future research may need to pursue. Furthermore across the preceding sections a pattern seems to emerge with regard to *how*, not merely *what*, students experience in their first encounters with advanced mathematical topics, whether at school or at university. Fundamental to addressing issues of transition seems also to be the coordination and dialogue across educational levels – here mostly secondary and tertiary – and our survey revealed that at the moment this appears largely absent.

ACKNOWLEDGEMENT

We would like to acknowledge the assistance of Michèle Artigue and the group "Enseignement supérieur" of the Laboratory André Revuz at the University Paris 7, including members from other universities, in the preparation of this report.

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