



# Structure-Preserving Cryptography

(Invited talk in Asiacrypt 2015)

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# Modular Design in General



(mostly from Wikipedia)

## **Modular design:**

A design approach that subdivides a system into smaller parts that can be independently created and then used in different systems.



# Modular Design in General



Innovative R&D by NTT

## **Benefit:**

Reduction in cost

less customization,  
shorter learning time



# Modular Design in General



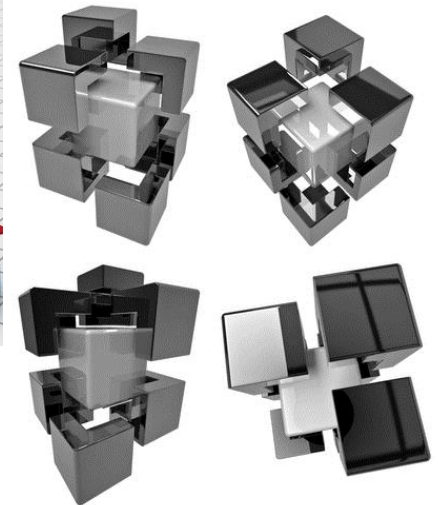
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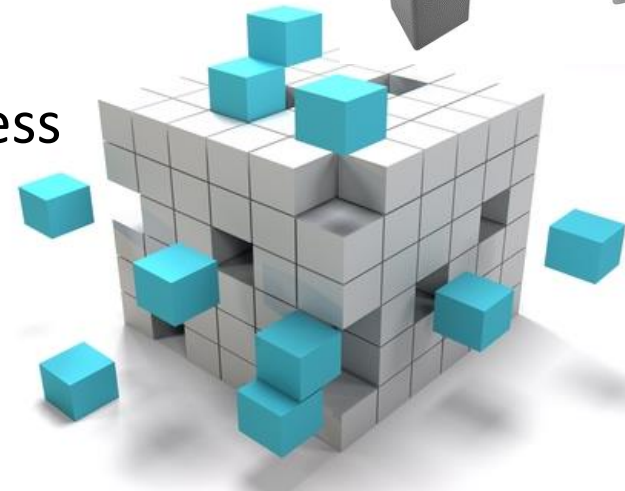
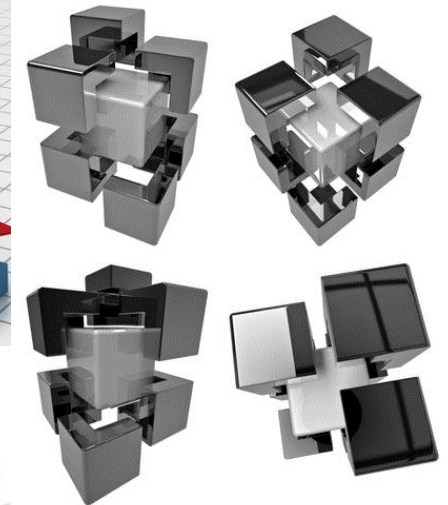
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Flexibility in design

Augmentation

adding new solution by merely  
plugging in a new module,  
making the manufacturing process  
more adaptive to change

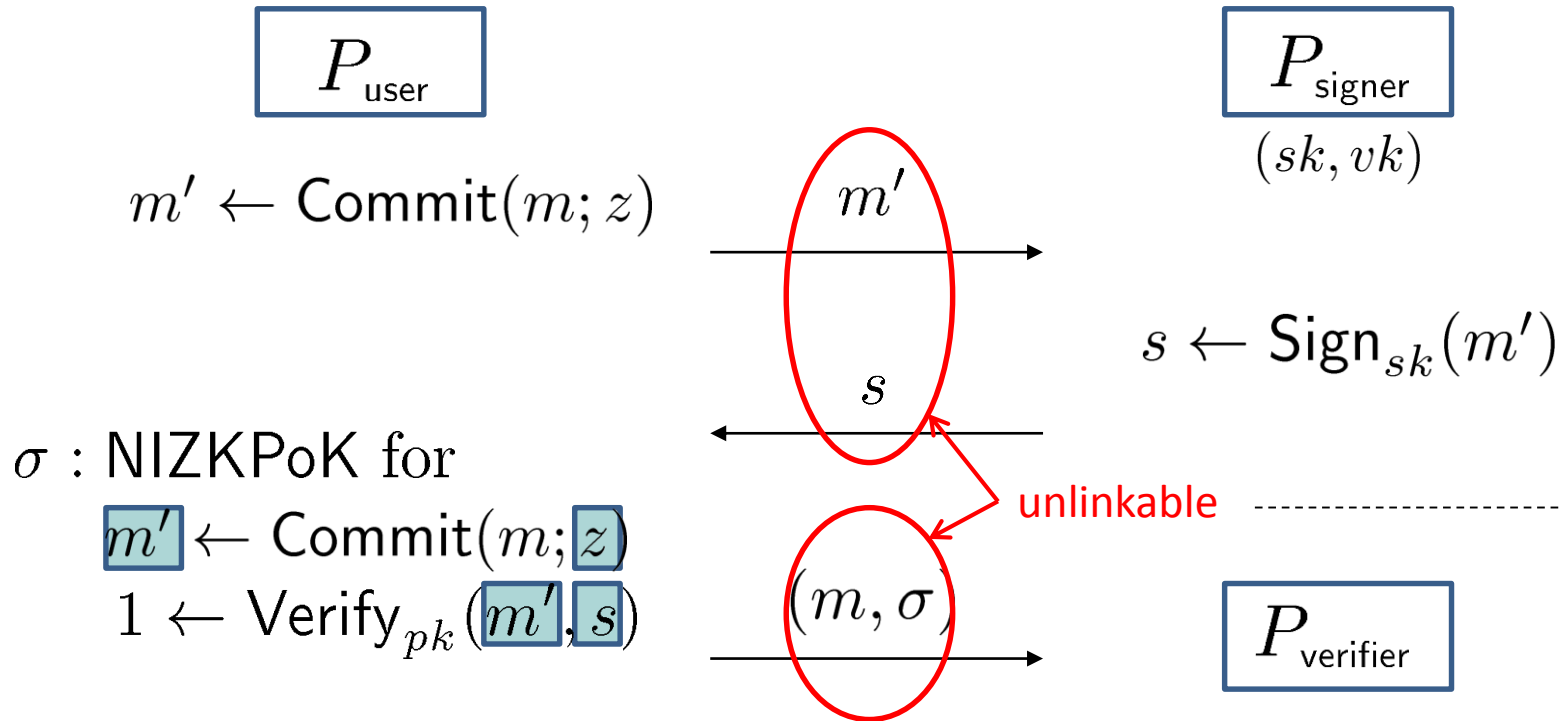


## **Downside:**

Low quality modular systems are not optimized for performance. This is usually due to the cost of putting up interfaces between modules.



# Blind Signatures [Fis06]





**Generic construction** in cryptography is a design approach that constructs a cryptographic system by combining smaller and abstract cryptographic primitives that conform to independent security notions.

## **Benefit:**

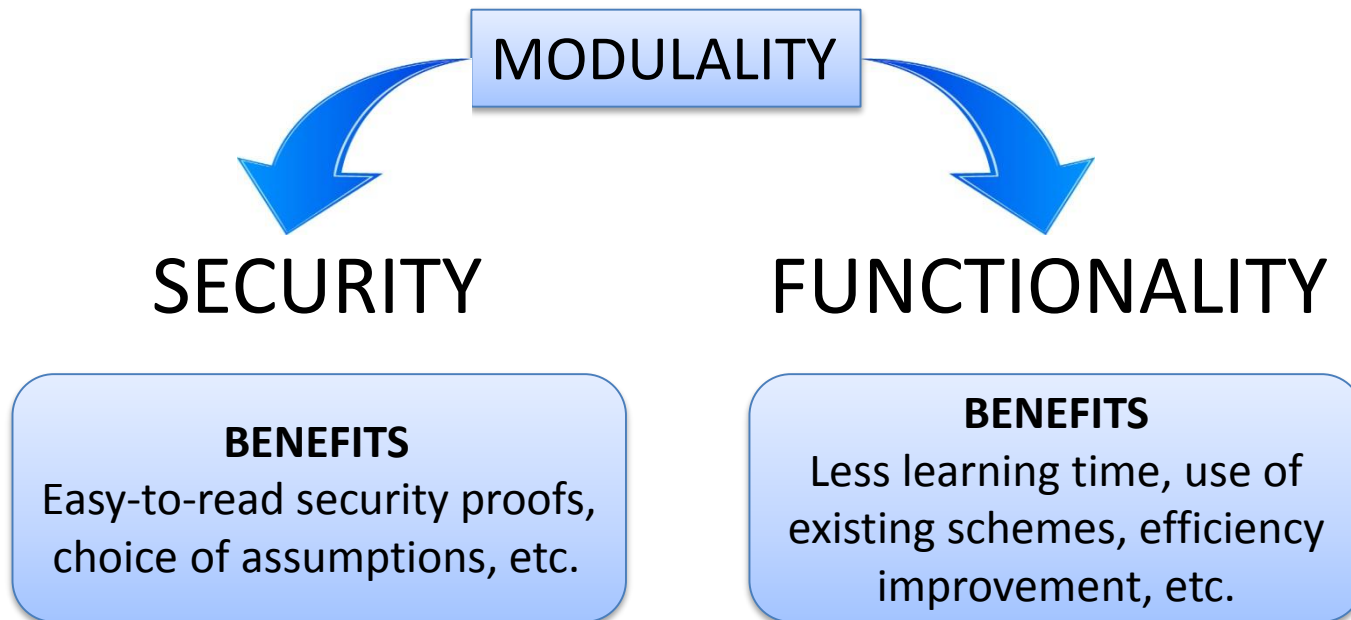
Reduction in cost: Simpler, easy-to-follow security proofs.

Flexibility in design: Off-the-shelf building blocks. Choice of assumptions.

Augmentation: New solution by plugging in a new building block

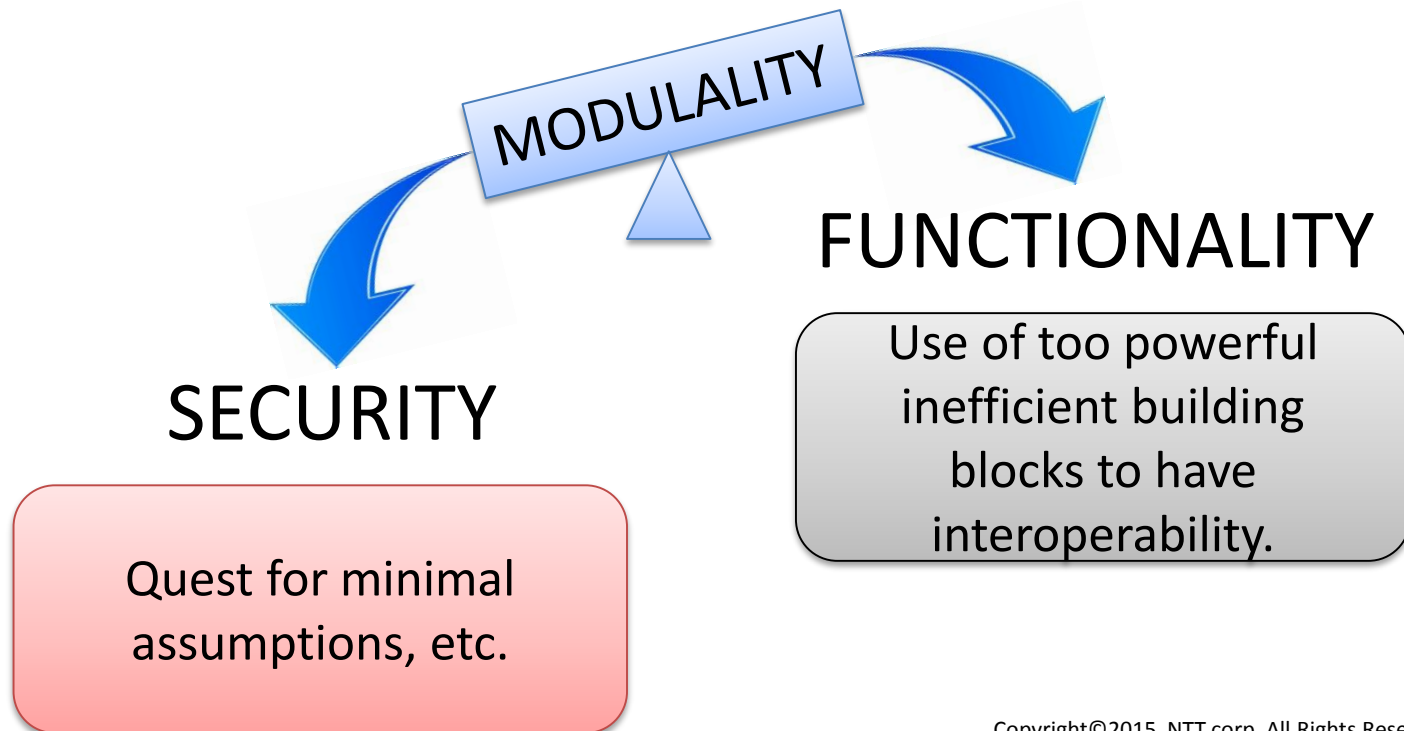


**Generic construction** in cryptography is a design approach that constructs a cryptographic system by combining smaller and abstract cryptographic primitives that conform to independent security notions.



## Downside:

Mainly used to show feasibility under minimal assumptions, or to show the underlying ideas. Often hard or ignored to find an efficient instantiation.



**Structure-Preserving Cryptography** is a framework for efficiently instantiating generic constructions using bilinear groups as a common ground for building blocks.

## Bilinear Groups

$$\Lambda = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, G_1, G_2)$$

$\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T$  : groups of prime order  $p$

$G_1, G_2, e(G_1, G_2)$  generate  $\mathbb{G}_1, \mathbb{G}_2$ , and  $\mathbb{G}_T$ , respectively

$$e : \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$$

$$\forall X \in \mathbb{G}_1, \forall Y \in \mathbb{G}_2, \forall a, b \in \mathbb{Z}_p, e(X^a, Y^b) = e(X, Y)^{ab}$$

A cryptographic scheme is structure-preserving if:

- (Group elements as interface) All public objects such as public-keys, messages, commitments, etc, merely consist of elements in  $\mathbb{G}_1$  and  $\mathbb{G}_2$ .

- (Correctness via PPEs) Verifying relations of interest can be done only by group operations, membership testing, and evaluating pairing product equations of the form

$$\prod_i^n e(A_i, Y_i) \prod_i^m e(X_i, B_i) \prod_i^m \prod_j^n e(X_i, Y_j)^{c_{ij}} = T.$$

where  $c_{ij}$  and  $T$  are system-defined constants.

# Structure-Preserving Schemes



A cryptographic scheme is structure-preserving if:

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All public data (keys, messages, commitments, etc, merely in  $\mathbb{G}_1$  and  $\mathbb{G}_2$ .

Aim for high interoperability and use of efficient Groth-Sahai proof system.

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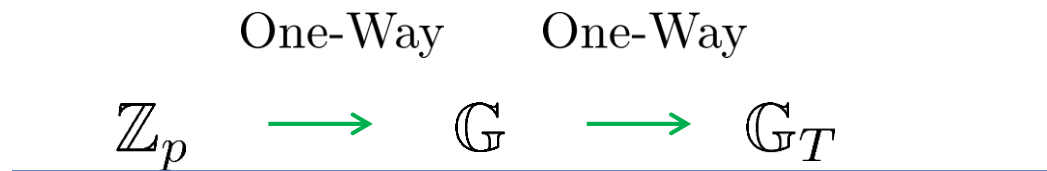
where  $c_{ij}$  and  $T$  are system-defined constants.

Constraints that make it hard to build efficient building blocks.

# Difficulty in Designing SP Primitives



## Example: Signature Scheme



Key generation:  $sk \longrightarrow vk$

Signing:  $m, r, sk \longrightarrow \sigma$

Verification:  $m \xrightarrow{\quad} vk$   
 $\sigma$

Must find a one-way structure within the same group (without hash functions).

# (In)Feasibility of SP Primitives



## Feasible

- NIWI, NIZK, CCA PKE, Non-shrinking Commitments, Shrinking TCR Commitments, (One-time/ Homomorphic/ Automorphic/ Equivalence Class) Signatures, Oblivious Transfer

## Unknown

- ID-based Encryption, Functional Encryption, ...

## Infeasible

- Unique signatures, (V)PRF, Deterministic Encryption, Shrinking CR Commitments



# Known Structure-Preserving Primitives



## Proof Systems

- NIWI, NIZK [Gro06, GS08, GSW10, EG14]
- Properties of GS-proofs [BCCKLS09, Fuc11, CKLM12]
- Simulation-Sound NIZK [Gro06, CCS08, HJ12]

## Signatures

- Constructions [Gro06, GH08, CLY09, AFGHO10, AHO10, AGHO11, CK11, ACD+12, CDH12, CK12, ADKNO13, LPJY13, ALP13, AGOTia14, AGOTib14, HS14, LJ14, CM14, BFFSST15, LPY15, AKOTi15, KPW15, Groth15]
- Bounds [AGHO11, AGO11, AGOTia14, AGOTib14]

## Public-Key Encryption

- CPA [ElG85, HK07, Sha07]
- CCA2 [CHKLN11]

## Commitments

- Constructions [Gro09, AFGHO10, AHO10, AHO12, AKOTi15]
- Bounds [AHO12]

## Oblivious Transfer

- Construction [DDvMNP15]

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Innovative R&D by NTT

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# Structure-Preserving Signatures

- The public keys, messages, and signatures consist of elements of  $\mathbb{G}_1$  and  $\mathbb{G}_2$ , and
- signature verification is done only by group operations, membership testing, and evaluation of pairing product equations.

# Advances of Research on SPS



Scheme	Assumptions	Signature Size	Group Type	Notes
[Fuc09]	$q$ -DHSDH	$21k + 11$	I	
[CLY09]	$q$ -HSDH, Flex-DH, DLIN	$9k + 4$	I	
[AFGHO10]	$q$ -SFP	7	Any	Unilateral
[AGHO11]	$q$ -type, SXDH	6	III	
		4	III	Unilateral
	interactive, SXDH	3	III	Unilateral
[AGOTi <sub>a</sub> 14]	interactive	3	Any	$k = 1$
[AGOTi <sub>b</sub> 14]	interactive	3	III	$k = 1$
	interactive	2	II	$k = 1$
[Gro06]	DLIN	$\mathcal{O}(k)$	I	
[CK12]	DLIN	$24k + 100 + 9x$	I	
	SXDH, RCDH	$18k + 77 + 6x$	III	Unilateral
[CDH12]	DLIN	$6k + 53$	I	
[ACDKNO12]	DLIN	17	I	
	SXDH, XDLIN	14	III	
		11	III	Unilateral
[ADKNO13]	DLIN	14	I	
[KPW15]	2-LIN	10	I	
	SXDH	7	III	Unilateral

# Static v.s. q-Type Assumptions



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## Static assumptions

- Simple as DLIN, SXDH
- Widely used.

Security of [ADKNO13, KPW15] is reduced to static assumptions with loss factor of  $1/q$  and  $1/q^2$

Security of [AFGHO10, AGHO11] is tightly reduced to q-type assumptions.

## q-Type Assumptions

- Consists of huge number of group elements
- Some are widely used, some are just ad-hoc.

## Static Assumptions

- Simple as DLIN, SXDH
- Widely used.
- Tight generic security

$$\text{CDH: } \mathcal{O}(\ell^2)/|\mathbb{G}|$$

Any generic adversary after  $\ell$  steps wins at most with this probability.

## q-Type Assumptions

- Consist of huge number of group elements
- Some are widely used, some are just ad-hoc.
- Loose generic security?

$$q\text{-SDH: } \mathcal{O}(q \ell^2)/|\mathbb{G}|$$

Given:  $G, G^x, G^{x^2}, \dots, G^{x^q}$   
Find:  $G^{\frac{1}{x+c}}, c$

# q-Type Assumption: Example



## Simultaneous Flexible Pairing Assumption (SFP) [AFGHO10]

Given:  $(A, A', B, B', G_z, H_z, G_r, H_u) \in \mathbb{G}^8$  and

$(Z_i, R_i, S_i, T_i, U_i, V_i, W_i) \in \mathbb{G}^7$  for  $i = 1, \dots, q$  constrained that

$$e(A, A') = e(G_z, Z_i) e(G_r, R_i) e(S_i, T_i)$$

$$e(B, B') = e(H_z, Z_i) e(H_u, U_i) e(V_i, W_i)$$

Find:  $(Z, R, S, T, U, V, W)$  with  $Z \notin \{Z_1, \dots, Z_q\}$  and  $Z \neq 1$

# Tight Generic Hardness of SFP



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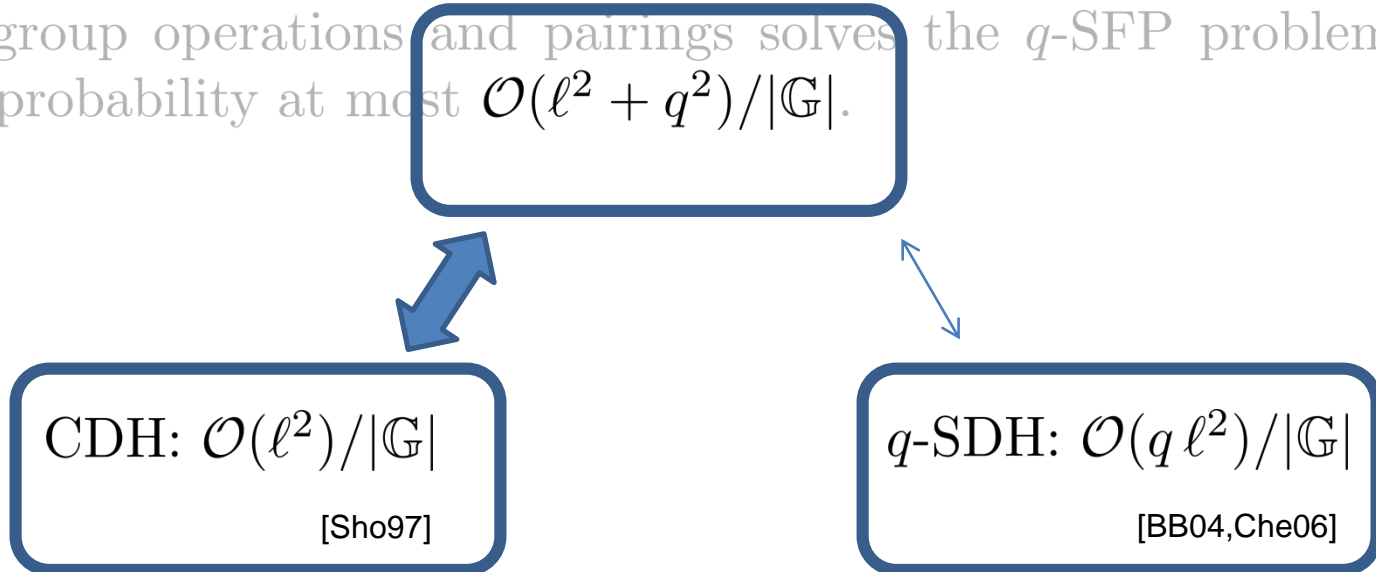
**Theorem.**[AGFHO15] Any generic algorithm  $\mathcal{A}$  performing up to  $\ell$  group operations and pairings solves the  $q$ -SFP problem with probability at most  $\mathcal{O}(\ell^2 + q^2)/|\mathbb{G}|$ .



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## Proof intuition:

Recall that reference answers satisfy

$$\begin{aligned}e(A, A') &= e(G_z, Z_i) e(G_r, R_i) e(S_i, T_i) \\e(B, B') &= e(H_z, Z_i) e(H_u, U_i) e(V_i, W_i).\end{aligned}$$

Taking the discrete log wrt  $G_r$ , the relations are written as

$$\begin{aligned}a a' &= g_z z_i + 1 r_i + s_i t_i \\b b' &= h_z z_i + h_u u_i + v_i w_i.\end{aligned}$$

## Proof intuition (cont'd):

The  $j$ -th group element viewed by the generic adversary is represented by a function  $F_j$  of a linear combination of variables  $a, a', b, b', g_z, 1, h_z, h_u, z_i, r_i, s_i, t_i, u_i, v_i, w_i$  where

$$r_i := a a' - g_z z_i - s_i t_i$$

$$u_i := (b b' - h_z z_i - v_i w_i) / h_u.$$

For every  $j, j' < \ell + q$ ,  $F_j - F_{j'}$  and  $F_j \cdot F_{j'}$  are Laurent polynomials with total degree of at most some small constants. Thus any of them vanishes at a random assignment to the variables only with probability at most  $2C_{\ell+q}(\text{const}/|\mathbb{G}|) = \mathcal{O}(\ell^2 + q^2)/|\mathbb{G}|$  as claimed.

# Lower Bounds on Signature Size



Group type	Messages	Lower bounds		Upper Bounds		
		Interactive	Non-interactive	Interactive	Non-interactive	
					$q$ -type	Static
Type-III	Unilateral	3 [AGHO11]	4 [AGO11]	3 [AGHO11]	4 [AGHO11]	7 [KPW15]
	Bilateral	3 [AGHO11]		3 [AGHO11]	6 [AGHO11]	14 [ACDKNO12]
Type-II	$M \in \mathbb{G}_1$ , Bilateral	3 [AGOTib14]		$3^{*1}$ [AGOTia14]		
	$M \in \mathbb{G}_2$	2 [AGOTib14]		$2^{*2}$ [AGOTib14]	3 [AGOTib14]	
Type-I	N/A	3 [AGOTia14]		$3^{*2}$ [AGOTia14]	7 [AFGHO10]	10 [KPW15]

\*1: Single-element message. Vector message possible.

\*2: Single-element message.

## Linearly homomorphic SPS [LPJY13]

- Application to Quasi-adaptive NIZK

## Selectively randomizable SPS [AGOTi14a]

- Flexibly change signatures from strongly unforgeable to randomizable ones

## SPS for equivalence classes [HS14,FHS15]

- Can sign on equivalence classes defined by vector of group elements
- Application to optimal-round blind signatures without using GS-proofs

## Fully SPS [AKOTi15,Gro15(tomorrow!)]

- Even secret-keys are group elements

# Open Problems on SPS



- Find more lower bounds for the case of non-interactive assumptions.
- Separately show lower bounds for static assumptions. Are they different from those for  $q$ -type assumptions?
- Show constant-size SPS with a tight reduction to simple assumptions.



# Structure-Preserving Commitments

- The commitment keys, messages, commitments, and opening informations consist of elements of  $\mathbb{G}_1$  and  $\mathbb{G}_2$ , and
- opening verification is done only by group operations, membership testing, and evaluation of pairing product equations.

## Performance of Structure-Preserving Commitment Schemes

Group	Scheme	msg	key	com	open	#(PPE)	asmpt.
Type-I	[Gro09]	$k$	$2k + 4$	$2_{(\mathbb{G}_T)}$	2	2	STP
	[AHO10]	$k$	$2k + 2$	$2_{(\mathbb{G}_T)}$	2	2	SDP
	[CLY09]	$k$	5	$3k$	$3k$	$3k$	DLIN
	[AHO10]	$k$	$2k + 2$	$2k + 2$	2	2	SDP
	[AHO12]	$k$	$2k + 3$	$k + 2$	2	2	SDP
Type-III	[AHO12]	$(0, k)$	$(k + 1, 1)$	$(1, k)$	$(0, 1)$	1	DBP



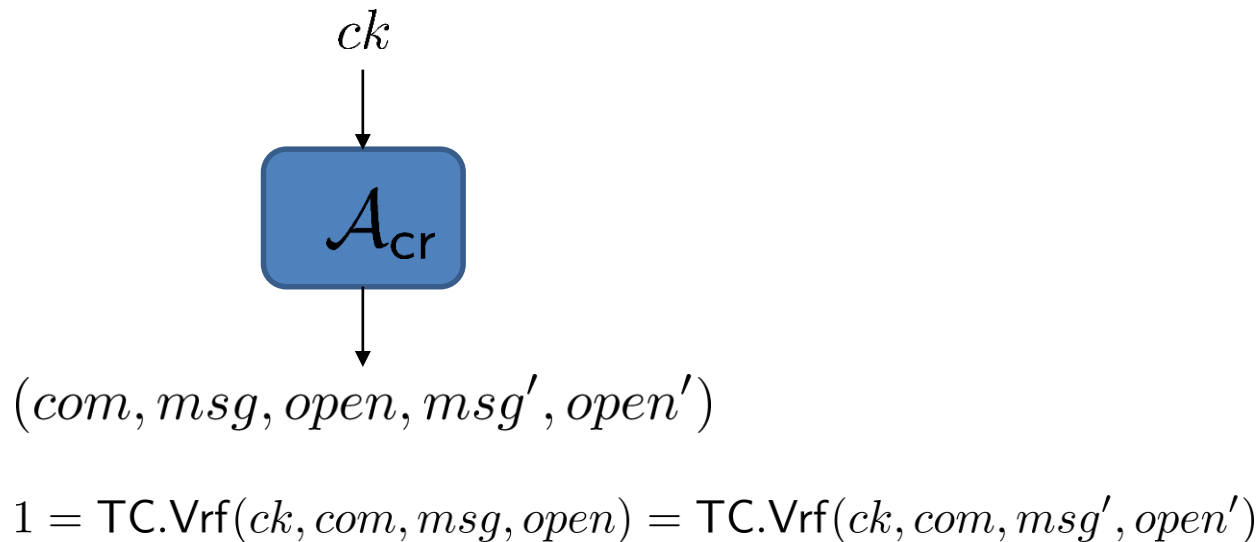
$$|msg| < |com|$$

Abe, Haralambiev, Ohkubo, “Group to Group Commitments Do Not Shrink”, Eurocrypt 2012



**Theorem 9 [AHO12].** If the discrete-logarithm problem is hard in the base groups, key generation and commitment algorithms are algebraic, and  $|com| < |msg|$ , then the commitment scheme is not binding.

## Binding Property (Collision Resistance)

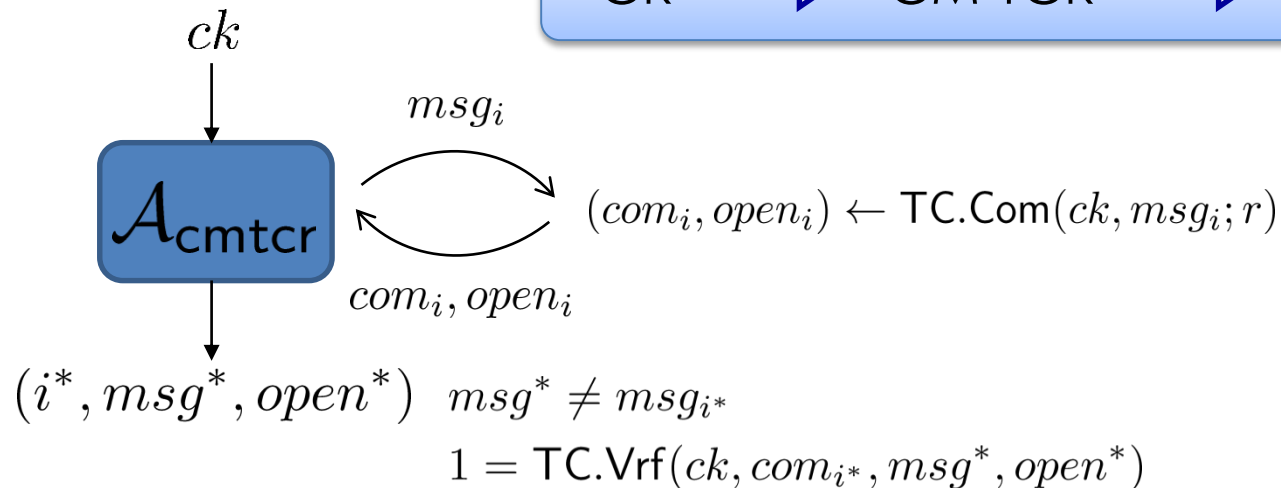


# SPCs Do Shrink, Sometimes



**Theorem [AKOTi14].** There exists a shrinking homomorphic trapdoor structure-preserving commitment scheme that is chosen message target collision resistant (CMTCR) if there exist a one-time non-adaptive chosen message secure structure-preserving partially one-time signature scheme (POS) and a  $\gamma$  target collision resistant trapdoor commitment scheme ( $\gamma$ -TC) exists.

## Notion of CMTCR



# Impossibility Argument for CR



If TC.Com is algebraic, commitment  $\vec{C}$  and opening  $\vec{D}$  are computed by linear combination of message  $\vec{M}$  and commitment key  $\vec{V}$ . Coefficient matrix  $B$  may depend on  $\vec{M}$ ,  $\vec{V}$  and internal random coins.

$$\begin{pmatrix} \vec{C} \\ \vec{D} \end{pmatrix} = \begin{pmatrix} B_1 & B_2 \\ B_3 & B_4 \end{pmatrix} \begin{pmatrix} \vec{M} \\ \vec{V} \end{pmatrix}$$

If  $|\vec{C}| < |\vec{M}|$ , there exists  $\vec{M}' (\neq \vec{M})$  that  $\vec{C} = (\vec{B}_1 | \vec{B}_2)(\vec{M}' | \vec{V})^T$ .

To compute such  $\vec{M}'$ , matrix  $\vec{B}$  must be known to the adversary.

# Impossibility Argument for CR



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In the CR game, it is the adversary that chooses the random coins. Hence it knows matrix B.

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In the CR game, it is the adversary that chooses the random coins. Hence it knows matrix B.

In CM-TCR game, it is the challenger that chooses the random coins. Thus matrix B is unknown to the adversary.



The impossibility argument does not apply to CM-TCR.

If  $|\vec{C}| < |\vec{M}|$ , there exists  $\vec{M}' (\neq \vec{M})$  that  $\vec{C} =$   
 To compute such  $\vec{M}'$ , matrix  $\vec{B}$  must be known

# Shrinking SPTC : Generic Construction



## Building blokcs

- Partial one-time signature scheme (POS)
- $\gamma$ -target collision resistant commitment scheme ( $\text{TC}_\gamma$ )

$\text{Com}(ck, msg)$

1.  $(M^{(1)}, \dots, M^{(\kappa)}) \in \mathcal{M}_{\text{pos}}^\kappa \leftarrow msg$

Divide message into k blocks

2.  $(vk_{\text{pos}}, sk_{\text{pos}}) \leftarrow \text{POS.Key}(gk)$

Generate one-time key

3. For  $i = 1, \dots, \kappa$

$$(ovk_{\text{pos}}^{(i)}, osk_{\text{pos}}^{(i)}) \leftarrow \text{POS.Ovk}(gk)$$

Sign each message block

$$\sigma_{\text{pos}}^{(i)} \leftarrow \text{POS.Sign}(sk_{\text{pos}}, osk_{\text{pos}}^{(i)}, M^{(i)})$$

4.  $(com_{\text{gbc}}, open_{\text{gbc}}) \leftarrow \text{TC}_\gamma.\text{Com}(sk_{\text{gbc}}, osk_{\text{gbc}}^{(1)}, \dots, osk_{\text{pos}}^{(\kappa)})$

Commit to the one-time keys

5.  $com := com_{\text{gbc}}$

$$open := (open_{\text{gbc}}, vk_{\text{pos}}, ovk_{\text{pos}}^{(1)}, \dots, ovk_{\text{pos}}^{(\kappa)}, \sigma_{\text{gbc}}^{(1)}, \dots, \sigma^{(\kappa)})$$

# Shrinking SPTC : sketch



Concrete SPTC from POS and  $TC_\gamma$  in Type-III groups

$$\text{Commitment-key : } ck = \left( G, \tilde{G}, \tilde{X}, \tilde{X}_1, \dots, \tilde{X}_k, \tilde{Y}_1, \dots, \tilde{Y}_\ell \right)$$

$$\text{Message : } M = (\tilde{M}_1, \dots, \tilde{M}_{k \cdot \ell})$$

$$\text{Commitment : } C = \tilde{G}_u$$

$$\text{Opening : } D = \left( R, \{ \tilde{Z}_j, \tilde{R}_j \}_{j=1}^k, A_1, \dots, A_k, G_z, G_1, \dots, G_\ell \right)$$

Verification( $ck, M, C, D$ )

$$e(G, \tilde{G}_u) = e(R, \tilde{G}) e(G_z, \tilde{X}) \prod_{i=1}^k e(A_i, \tilde{X}_i) \prod_{i=1}^{\ell} e(G_i, \tilde{Y}_i)$$

for  $j = 1, \dots, k$

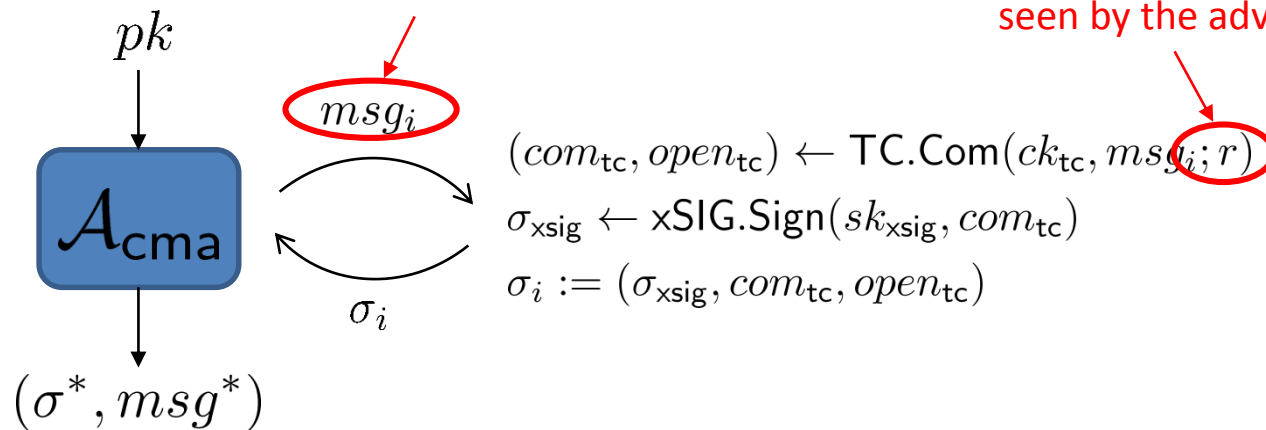
$$e(A_j, \tilde{G}) = e(G_z, \tilde{Z}_j) e(G, \tilde{R}_j) \prod_{i=1}^{\ell} e(G_i, \tilde{M}_{(j-1)\ell+i})$$

Message:	$k \cdot \ell$
Commit key:	$1 + k + \ell$
Trapdoor:	$1 + k + \ell$ (in $\mathbb{Z}_p$ )
Commitment:	1
Opening info:	$2 + 3k + \ell$
# of PPEs:	$1 + k$

# Usefulness of CM-TCR

CM-TCR is still useful in constructing CMA-secure signature schemes.

CMA-security Game



$$msg^* \notin \{msg_i\}$$

$$1 \leftarrow SIG.Vrf(pk, msg^*, \sigma^*)$$



A large, light blue, rounded shape on the left side of the slide, resembling a stylized drop or a partial circle.

# **Applications**

# Works on Structure-Preserving Crypto



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## Proof Systems

- NIWI, NIZK [Gro06, GS08, GSW10, EG14]
- Properties of GS-proofs [BCCKLS09, Fuc11, CKLM12]
- Simulation-Sound NIZK [Gro06, CCS08, HJ12]

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- Bounds [AGHO11, AGO11, AGOTia14, AGOTib14]

## Public-Key Encryption

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- CCA2 [CHKLN11]

## Commitments

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- Bounds [AHO12]

## Oblivious Transfer

- Construction [DDvMNP15]

## Applications(Blind signature, Group signature, Credential system, etc,...)

- [AFGHO10, CHKLN11, Kris11, ALP12, LPY12, HJ12, CKLM12, FKMV12, AJ13, BFG13, LPJY13, KR13, CMA13, SEHKMO13, ZLG13, ACDN14, LJYP14, LPJM14, AEHS14, LPDW14, ABGSS14, HRS15, FHS15, KM15, Ghada15]

# General Idea for Group Signatures



[AFGHO10] Signatures  
[BB04] one-time Signatures

Groth-Sahai Proof  
System

Encryption/  
Anonymous Tag  
System

“Group/Traceable Signature” = “Signature” + “Revocation Mechanism”

- Guarantees integrity of messages.
- Authenticate the signer.

- Opening, Tracing
- Claiming, Denial

Glued by NIZK that guarantees  
correct computation while  
hiding privacy related objects in  
each part.

# Comparison



Scheme	Construction Type	Claim & Trace	Deny Function	Anonymity w/ Trace	Anonymity Level	Concurrent Join	Sig. Size
[AHO10]	generic	no	no	no	CCA	yes	58
[LY09]	Tailor-made	yes	no	no	CPA	no	83
[ACHO11]	generic	yes	yes	yes	CCA	yes	107

Table: Summary of properties among group signature and traceable signature schemes that provide non-frameability (the signature size counts the number of group elements).



**Wrap Up**

- There are proof systems, signatures, encryption, and commitments over bilinear groups that are structure-preserving and thus interoperable each other.
- They can be used for modular construction of intricate cryptographic tasks. And the efficiency of the resulting scheme can be evaluated with concrete figures.
  - There is room for hand-crafted optimization by carefully choosing which elements are hidden and which are put in the clear.
- There are interesting open problems both in practice (efficiency improvements) and theory (lower bounds), and missing important tools like IBE, FE, etc.

## Authors of papers that study or use SP primitives.

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