From Quantum Chaos
To
Anderson Localization

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Columbia University
NEC Laboratories America
$N \times N$ matrices with random matrix elements. $N \to \infty$

**Dyson Ensembles**

<table>
<thead>
<tr>
<th>Matrix elements</th>
<th>Ensemble</th>
</tr>
</thead>
<tbody>
<tr>
<td>real</td>
<td>orthogonal</td>
</tr>
<tr>
<td>complex</td>
<td>unitary</td>
</tr>
<tr>
<td>$2 \times 2$</td>
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$E_\alpha$ - spectrum (set of eigenvalues)

$\delta_1 \equiv \langle E_{\alpha+1} - E_\alpha \rangle$ - mean level spacing
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\[ \delta_1 \equiv \langle E_{\alpha+1} - E_\alpha \rangle \quad \text{- mean level spacing} \]

\[ \langle \ldots \ldots \rangle \quad \text{- ensemble averaging} \]
\( E_\alpha \) - spectrum (set of eigenvalues)

\[ \delta_1 \equiv \langle E_{\alpha+1} - E_\alpha \rangle \]

\( \langle \ldots \ldots \rangle \) - mean level spacing

\[ s \equiv \frac{E_{\alpha+1} - E_\alpha}{\delta_1} \]

\( s \) - ensemble averaging

- spacing between consecutive eigenvalues
RANDOM MATRIX THEORY

$E_\alpha$ - spectrum (set of eigenvalues)

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$P(s)$ - distribution function
RANDOM MATRIX THEORY

$E_\alpha$

- spectrum (set of eigenvalues)

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- mean level spacing

$\langle \ldots \rangle$

- ensemble averaging

$s \equiv \frac{E_{\alpha+1} - E_\alpha}{\delta_1}$

- spacing between consecutive eigenvalues

$P(s)$

- distribution function

Spectral Rigidity

Level repulsion

$P(s = 0) = 0$

$P(s \ll 1) \propto s^\beta \quad \beta = 1, 2, 4$
Noncrossing rule (theorem) \[ P(s = 0) = 0 \]

Suggested by Hund (Hund F. 1927 Phys. v.40, p.742)


Usually textbooks present a simplified version of the justification due to Teller (Teller E., 1937 J. Phys. Chem 41 109).

Arnold V. I., 1972 Funct. Anal. Appl.v. 6, p.94

In general, a multiple spectrum in typical families of quadratic forms is observed only for two or more parameters, while in one-parameter families of general form the spectrum is simple for all values of the parameter. Under a change of parameter in the typical one-parameter family the eigenvalues can approach closely, but when they are sufficiently close, it is as if they begin to repel one another. The eigenvalues again diverge, disappointing the person who hoped, by changing the parameter to achieve a multiple spectrum.

\[ \hat{H}(x) \Rightarrow E_\alpha(x) \]
In general, a multiple spectrum in typical families of quadratic forms is observed only for two or more parameters, while in one-parameter families of general form the spectrum is simple for all values of the parameter. Under a change of parameter in the typical one-parameter family the eigenvalues can approach closely, but when they are sufficiently close, it is as if they begin to repel one another. The eigenvalues again diverge, disappointing the person who hoped, by changing the parameter to achieve a multiple spectrum.
1. The assumption is that the matrix elements are statistically independent. Therefore probability of two levels to be degenerate vanishes.

2. If $H_{12}$ is real (orthogonal ensemble), then for $s$ to be small two statistically independent variables ($H_{22} - H_{11}$ and $H_{12}$) should be small and thus $P(s) \propto s \beta = 1$
1. The assumption is that the matrix elements are statistically independent. Therefore probability of two levels to be degenerate vanishes.

2. If $H_{12}$ is real (orthogonal ensemble), then for $s$ to be small two statistically independent variables $(H_{22} - H_{11})$ and $H_{12}$ should be small and thus $P(s) \propto s$ \hspace{1cm} $\beta = 1$

3. Complex $H_{12}$ (unitary ensemble) both $Re(H_{12})$ and $Im(H_{12})$ are statistically independent three independent random variables should be small $P(s) \propto s^2$ \hspace{1cm} $\beta = 2$
Poisson – completely uncorrelated levels

Wigner-Dyson; GOE

Poisson

Gaussian Orthogonal Ensemble

Orthogonal $\beta=1$

Unitary $\beta=2$

Simplectic $\beta=4$
\( N \times N \) matrices with random matrix elements. \( N \to \infty \)

### Spectral Rigidity

- **Level repulsion**
  
  \[ P(s << 1) \propto s^\beta \]
  
  \[ \beta = 1, 2, 4 \]

### Dyson Ensembles

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<th>Matrix elements</th>
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<th>( \beta )</th>
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<tr>
<td>real</td>
<td>orthogonal</td>
<td>1</td>
<td>T-inv potential</td>
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<tr>
<td>complex</td>
<td>unitary</td>
<td>2</td>
<td>broken T-invariance (e.g., by magnetic field)</td>
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<tr>
<td>2 \times 2 matrices</td>
<td>simplectic</td>
<td>4</td>
<td>T-inv, but with spin-orbital coupling</td>
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Finite size quantum physical systems

Atoms
Nuclei
Molecules

Quantum Dots
Main goal is to classify the eigenstates in terms of the quantum numbers.

For the nuclear excitations this program does not work.

N. Bohr, Nature 137 (1936) 344.
Main goal is to classify the eigenstates in terms of the quantum numbers

For the nuclear excitations this program does not work

E.P. Wigner

Study spectral *statistics* of a *particular* quantum system - a given nucleus
Particular nucleus

$^{166}$Er

Spectra of several nuclei combined (after spacing) rescaling by the mean level

N. Bohr, Nature 137 (1936) 344.
Main goal is to classify the eigenstates in terms of the quantum numbers

For the nuclear excitations this program does not work

**E.P. Wigner**

Study spectral statistics of a particular quantum system - a given nucleus

<table>
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<th>Random Matrices</th>
<th>Atomic Nuclei</th>
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<tr>
<td>• Ensemble</td>
<td>• Particular quantum system</td>
</tr>
<tr>
<td>• Ensemble averaging</td>
<td>• Spectral averaging (over $\alpha$)</td>
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Nevertheless

Statistics of the nuclear spectra are almost exactly the same as the Random Matrix Statistics
T-invariance (CP) violation – crossover between Orthogonal and Unitary ensembles
Q: Why the random matrix theory (RMT) works so well for nuclear spectra
Why the random matrix theory (RMT) works so well for nuclear spectra

Original answer:
These are systems with a large number of degrees of freedom, and therefore the “complexity” is high.

Later it became clear that there exist very “simple” systems with as many as 2 degrees of freedom (d=2), which demonstrate RMT-like spectral statistics.
Classical Dynamical Systems with \( d \) degrees of freedom

**Integrable Systems**

The variables can be separated and the problem reduces to \( d \) one-dimensional problems

\( d \) integrals of motion
Classical ($\hbar = 0$) Dynamical Systems with $d$ degrees of freedom

### Integrable Systems

The variables can be separated and the problem reduces to $d$ one-dimensional problems

### Examples

1. A ball inside rectangular billiard; $d = 2$

   - Vertical motion can be separated from the horizontal one
   - Vertical and horizontal components of the momentum, are both integrals of motion
Classical ($\hbar = 0$) Dynamical Systems with $d$ degrees of freedom

**Integrable Systems**

The variables can be separated and the problem reduces to $d$ one-dimensional problems

**Examples**

1. A ball inside rectangular billiard; $d=2$
   - **Vertical** motion can be separated from the **horizontal** one
   - **Vertical** and **horizontal** components of the momentum, are both integrals of motion

2. Circular billiard; $d=2$
   - **Radial** motion can be separated from the **angular** one
   - **Angular** momentum and **energy** are the integrals of motion
<table>
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# Classical Dynamical Systems with $d$ degrees of freedom

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| Chaotic Systems | The variables **can not** be separated $\Rightarrow$ there is only one integral of motion - energy |

# Examples
Classical Dynamical Systems with $d$ degrees of freedom

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Rectangular and circular billiard, Kepler problem, . . . , 1d Hubbard model and other exactly solvable models, . .

**Chaotic Systems**
The variables can not be separated $\Rightarrow$ there is only one integral of motion - energy

**Examples**
Classical Dynamical Systems with $d$ degrees of freedom

**Integrable Systems**
- The variables can be separated $\Rightarrow d$ one-dimensional problems $\Rightarrow d$ integrals of motion
- Rectangular and circular billiard, Kepler problem, ..., 1d Hubbard model and other exactly solvable models, ...

**Chaotic Systems**
- The variables *can not* be separated $\Rightarrow$ there is only one integral of motion - energy

**Examples**
- Stadium
### Classical Dynamical Systems with $d$ degrees of freedom

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### Examples

Stadium
### Classical Dynamical Systems with \( d \) degrees of freedom

#### Integrable Systems

The variables can be separated \( \Rightarrow \) \( d \) one-dimensional problems \( \Rightarrow \) \( d \) integrals of motion

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#### Chaotic Systems

The variables \textbf{can not} be separated \( \Rightarrow \) there is only one integral of motion - energy

### Examples

- **Sinai billiard**
- **Stadium**
Classical Dynamical Systems with $d$ degrees of freedom

Integrable Systems
- The variables can be separated $\Rightarrow$ $d$ one-dimensional problems $\Rightarrow d$ integrals of motion
- Rectangular and circular billiard, Kepler problem, . . . , 1d Hubbard model and other exactly solvable models, . . .

Chaotic Systems
- The variables can not be separated $\Rightarrow$ there is only one integral of motion - energy

Examples
- Sinai billiard
- Stadium
- Kepler problem in magnetic field
Chaotic Systems

The variables *can not* be separated $\rightarrow$ there is only one integral of motion - energy.

**Examples**

- Sinai billiard
- Stadium
- Kepler problem in magnetic field

Yakov Sinai
Leonid Bunimovich
Johnnes Kepler
Classical Chaos
\[ \hbar = 0 \]

- Nonlinearities
- Lyapunov exponents
- Exponential dependence on the original conditions
- Ergodicity

Quantum description of any System with a finite number of the degrees of freedom is a linear problem – Schrodinger equation

Q: What does it mean Quantum Chaos?
$\hbar \neq 0$

Bohigas – Giannoni – Schmit conjecture

Characterization of Chaotic Quantum Spectra and Universality of Level Fluctuation Laws

O. Bohigas, M. J. Giannoni, and C. Schmit

Division de Physique Théorique, Institut de Physique Nucléaire, F-91406 Orsay Cedex, France

(Received 2 August 1983)

It is found that the level fluctuations of the quantum Sinai’s billiard are consistent with the predictions of the Gaussian orthogonal ensemble of random matrices. This reinforces the belief that level fluctuation laws are universal.

In summary, the question at issue is to prove or disprove the following conjecture: Spectra of time-reversal–invariant systems whose classical analogs are $K$ systems show the same fluctuation properties as predicted by GOE.
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What does it mean Quantum Chaos?

Two possible definitions

Chaotic classical analog

Wigner - Dyson-like spectrum
Wigner-Dyson

Classical

Integrable

Chaotic

Quantum

Poisson

Wigner-Dyson
Poisson to Wigner-Dyson crossover

Important example: quantum particle subject to a random potential – disordered conductor

Scattering centers, e.g., impurities
Important example: quantum particle subject to a random potential – disordered conductor

• Scattering centers, e.g., impurities

As well as in the case of Random Matrices (RM) there is a luxury of ensemble averaging.

• The problem is much richer than RM theory

• There is still a lot of universality.

Anderson localization (1956)

At strong enough disorder all eigenstates are localized in space
Correlations due to Localization in Quantum Eigenfunctions of Disordered Microwave Cavities

Prabhakar Pradhan and S. Sridhar
Department of Physics, Northeastern University, Boston, Massachusetts 02115
(Received 28 February 2000)

\( f = 3.04 \text{ GHz} \)

\( f = 7.33 \text{ GHz} \)

Anderson Insulator

Anderson Metal
Poisson to Wigner-Dyson crossover

Important example: quantum particle subject to a random potential – disordered conductor

• Scattering centers, e.g., impurities

Models of disorder: Randomly located impurities

\[ U(\vec{r}) = \sum_i u(\vec{r} - \vec{r}_i) \]
Poisson to Wigner-Dyson crossover

**Important example:** quantum particle subject to a random potential – disordered conductor

*Scattering centers, e.g., impurities*

**Models of disorder:**

- Randomly located impurities
- White noise potential

**Anderson model** – tight-binding model with **on-site** disorder

**Lifshits model** – tight-binding model with **off-diagonal** disorder

\[ U(\vec{r}) = \sum_i u(\vec{r} - \vec{r}_i) \]

\[ u(\vec{r}) \to \lambda \delta(\vec{r}) \quad \lambda \to 0 \quad c_{im} \to \infty \]
Anderson Model

- Lattice - tight binding model
- Onsite energies $\varepsilon_i$ - random
- Hopping matrix elements $I_{ij}$

$-W < \varepsilon_i < W$
uniformly distributed

$I_{ij} = \begin{cases} I & \text{if } i \text{ and } j \text{ are nearest neighbors} \\ 0 & \text{otherwise} \end{cases}$

Anderson Transition

$I < I_c$
Insulator
All eigenstates are **localized**
Localization length $\xi$

$I > I_c$
Metal
There appear states **extended** all over the whole system
Localization of single-electron wave-functions:

\[
\left[-\frac{\nabla^2}{2m} + U(r) - \epsilon_F\right] \psi_\alpha(r) = \xi_\alpha \psi_\alpha(r)
\]

\[\psi_\alpha(x)\] extended

\[p_F^{-1}\]

\[\xi_{loc}\]

localized
Localization of single-electron wave-functions:

\[
\left[-\frac{\nabla^2}{2m} + U(r) - \epsilon_F\right] \psi_\alpha(r) = \xi_\alpha \psi_\alpha(r)
\]

\[d=1; \text{ All states are localized}\]
\[d=2; \text{ All states are localized}\]
\[d>2; \text{ Anderson transition}\]
Anderson Transition

\[ I < I_c \]

**Insulator**

*All eigenstates are localized*

*Localization length* \( \xi \)

*The eigenstates, which are localized at different places will not repel each other*

*Poisson spectral statistics*

\[ I > I_c \]

**Metal**

*There appear states extended all over the whole system*

*Any two extended eigenstates repel each other*

*Wigner – Dyson spectral statistics*
Zharekeshev & Kramer.

**Exact diagonalization of the Anderson model**

\[ \left[ -\frac{\nabla^2}{2m} + WU(\vec{r}) - \varepsilon_\alpha \right] \psi_\alpha(\vec{r}) = 0 \]
Quantum particle in a random potential *(Thouless, 1972)*

**Energy scales**

1. **Mean level spacing**
   \[ \delta_1 = \frac{1}{\nu} \times L^d \]
   
   - \( \delta_1 \) is the mean level spacing.
   - \( L \) is the system size.
   - \( d \) is the number of dimensions.

2. **Thouless energy**
   \[ E_T = \frac{\hbar D}{L^2} \]
   
   - \( E_T \) has a meaning of the *inverse diffusion time* of the traveling through the system or the *escape rate* (for open systems).
   - \( \hbar \) is the reduced Planck constant.
   - \( D \) is the diffusion constant.

\[ g = \frac{E_T}{\delta_1} \]

- \( g \) is the Thouless conductance.

\[ g = \frac{\mathcal{G} \hbar}{e^2} \]
The same statistics of the random spectra and one-particle wave functions (eigenvectors)
Scaling theory of Localization

(Abrahams, Anderson, Licciardello and Ramakrishnan 1979)

\[ g = \frac{E_T}{\delta_1} \]

Dimensionless Thouless conductance

\[ g = \frac{G\hbar}{e^2} \]

\[ L = 2L = 4L = 8L \ldots \]

without quantum corrections

\[ E_T \propto L^{-2} \quad \delta_1 \propto L^{-d} \]
Scaling theory of Localization
(Abrahams, Anderson, Licciardello and Ramakrishnan 1979)

\[ g = \frac{E_T}{\delta_1} \]

Dimensionless \textit{Thouless} conductance

\[ g = \frac{G h}{e^2} \]

\[ L = 2L = 4L = 8L \ldots \]

without quantum corrections

\[ E_T \propto L^{-2} \quad \delta_1 \propto L^{-d} \]

\[ \frac{d(\log g)}{d(\log L)} = \beta(g) \]
$\beta$ - function

$$\frac{d \log g}{d \log L} = \beta(g)$$

Unstable fixed point

Metal – insulator transition in 3D
All states are localized for $d=1,2$
Conductance $g$
Anderson transition in terms of pure level statistics

![Graph showing level statistics with curves labeled as metal, W=5, critical, 16.5, insulator, 100, and distributions labeled as Wigner and Poisson.](image-url)
Integrable

All chaotic systems resemble each other.

Chaotic

All integrable systems are integrable in their own way.
Consider an integrable system. Each state is characterized by a set of quantum numbers.

It can be viewed as a point in the space of quantum numbers. The whole set of the states forms a lattice in this space.

A perturbation that violates the integrability provides matrix elements of the hopping between different sites (Anderson model!!?)
Consider an integrable system. Each state is characterized by a set of quantum numbers.

It can be viewed as a point in the space of quantum numbers. The whole set of the states forms a lattice in this space.

A perturbation that violates the integrability provides matrix elements of the hopping between different sites (Anderson model !?)

Weak enough hopping - Localization - Poisson
Strong hopping - transition to Wigner-Dyson

Q: Does Anderson localization provide a generic scenario for the Wigner-Dyson to Poisson crossover?
The very definition of the localization is not invariant - one should specify in which space the eigenstates are localized.

Level statistics is invariant:

- **Poissonian statistics** \( \exists \) basis where the eigenfunctions are localized
- **Wigner-Dyson statistics** \( \forall \) basis the eigenfunctions are extended
**Example 1**

**Doped semiconductor**

- **Low concentration of donors** → **Electrons are localized on donors** ⇒ **Poisson**

- **Higher donor concentration** → **Electronic states are extended** ⇒ **Wigner-Dyson**
Example 1

**Doped semiconductor**

- Low concentration of donors:
  - Electrons are localized on donors ⇒ Poisson
- Higher donor concentration:
  - Electronic states are extended ⇒ Wigner-Dyson

Example 2

**Rectangular billiard**

- Two integrals of motion:
  \[ p_x = \frac{\pi n}{L_x}; \quad p_y = \frac{\pi m}{L_x} \]
- Lattice in the momentum space
- Line (surface) of constant energy
- Ideal billiard
  - localization in the momentum space ⇒ Poisson
  - Lattice in the momentum space
- Deformation or smooth random potential
  - delocalization in the momentum space ⇒ Wigner-Dyson
Localization and diffusion in the angular momentum space

Diffusion and Localization in Chaotic Billiards

Fausto Borgonovi, Giulio Casati, Baowen Li

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4 Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, Pavia, Italy
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6 Department of Physics and Centre for Nonlinear and Complex Systems, Hong Kong Baptist University, Hong Kong
7 Center for Applied Mathematics and Theoretical Physics, University of Maribor, Križanova 2, 2000 Maribor, Slovenia

(Received 29 July 1996)

\[ \varepsilon \equiv \frac{a}{R} \]

\[ \varepsilon > 0 \text{ Chaotic stadium} \]

\[ \varepsilon \to 0 \text{ Integrable circular billiard} \]

Angular momentum is the integral of motion

\[ \hbar = 0; \quad \varepsilon \ll 1 \]

Diffusion in the angular momentum space

\[ D \propto \varepsilon^{5/2} \]
Localization and diffusion in the angular momentum space

\[ \varepsilon \equiv \frac{a}{R} \]

\[ \varepsilon > 0 \] Chaotic stadium

\[ \varepsilon \rightarrow 0 \] Integrable circular billiard

Angular momentum is the integral of motion

\[ \hbar = 0; \quad \varepsilon \ll 1 \]

Diffusion in the angular momentum space

\[ D \propto \varepsilon^{5/2} \]
1D Hubbard Model on a periodic chain

\[ H = t \sum_{i,\sigma} \left( c_{i,\sigma}^+ c_{i+1,\sigma} + c_{i+1,\sigma}^+ c_{i,\sigma} \right) + U \sum_{i,\sigma} n_{i,\sigma} n_{i,-\sigma} + V \sum_{i,\sigma,\sigma'} n_{i,\sigma} n_{i+1,\sigma'} \]

- \( V = 0 \): Hubbard model, integrable
- \( V \neq 0 \): extended Hubbard model, nonintegrable

Onsite interaction
n. neighbours interaction
1D Hubbard Model on a periodic chain

\[ H = t \sum_{i,\sigma} \left( c_{i,\sigma}^+ c_{i+1,\sigma} + c_{i+1,\sigma}^+ c_{i,\sigma} \right) + U \sum_{i,\sigma} n_{i,\sigma} n_{i,-\sigma} + V \sum_{i,\sigma,\sigma'} n_{i,\sigma} n_{i+1,\sigma'} \]

**Onsite interaction**

**n. neighbors interaction**

\( V = 0 \) Hubbard model **integrable**

\( V \neq 0 \) extended Hubbard model **nonintegrable**

12 sites
3 particles
Zero total spin
Total momentum \( \pi/6 \)

\( U=4 \) \( V=0 \)

\( U=4 \) \( V=4 \)
1D $t$-$J$ model on a periodic chain
1D $t$-$J$ model on a periodic chain

$J$ exchange

$t$ hopping

forbidden
1D $t$-$J$ model on a periodic chain

$N=16$; one hole

$J=t$

$J=2t$

$J=5t$

D. Poilblanc, T. Ziman, J. Bellisard, F. Mila & G. Montambaux

Europhysics Letters, v. 22, p. 537, 1993
Chaos in Nuclei – Delocalization?

1 2 3 4 5 6

Delocalization in Fock space

Fermi Sea