The Census of Edge-Transitive Tetravalent Graphs

SCDO, Queenstown,
MiGawdItsBeautiful,
New Zealand

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The Site

http://jan.ucc.nau.edu/~swilson/C4FullSite/index.html

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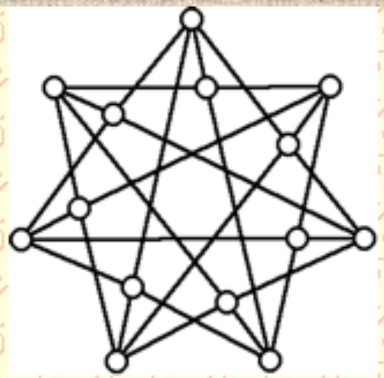


[Home] [Table] [Glossary] [Families]

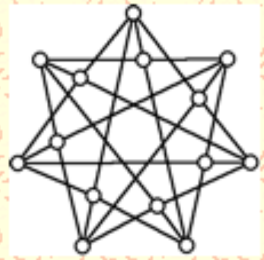
A Census of edge-transitive tetravalent graphs
The Full Census

Welcome to the Degree-4 census. This site will provide information about edge-transitive graphs of degree 4. This fourth edition shows graphs of up to 512 vertices. It is known to be incomplete in a few ways. We have chosen a limit of 512 vertices to reach and include Bouwer's generalization of the Gray Graph, and we invite your comments and contributions.

The main part of the census is the **Table** . This is a list of all graphs in the census, telling the name, the size of the symmetry group, girth, diameter and other information. Each line has a link to an individual page for that graph, with more information about the graph. A spreadsheet version of the Table is available. In either form, the Table is the place to browse for graphs of a certain Order.

Notation, definitions, vocabulary can be found in the **Glossary** . The Glossary is the textbook of the Census and contains all the careful descriptions of the graphs. This is not a paper Glossary, so use your browser's search function. If you meet a graph in the Table called "SDD(DG(F32))", and have no idea what that means, go to the Glossary and search for SDD, for DG and for F to find out. The Glossary also explains what notations on the summary pages tells you. Look in the Glossary to find out what Ivanov vectors or cyclic coverings are.

Summaries of the contributions of individual constructions is available at **Families** . Here, every construction of each



A Census of edge-transitive tetravalent graphs
Full Census Table

[N,i]	V	E	Tr	W?	B?	|AG|	vs	ds	#STO	gi	dm	NAME
C4[5, 1]	5	10	DT	W	NB	120	24	6	0	3	1	K5
C4[6, 1]	6	12	DT	U	NB	48	8	2	1	3	2	Octahedron
C4[8, 1]	8	16	DT	U	Bip	(2^7)(3^2)	144	36	2	4	2	K_4,4
C4[9, 1]	9	18	DT	W	NB	72	8	2	1	3	2	DW(3,3)
C4[10, 1]	10	20	DT	U	NB	320	32	8	1	4	2	W(5,2)
C4[10, 2]	10	20	DT	W	Bip	240	24	6	1	4	3	C_10(1,3)
C4[12, 1]	12	24	DT	U	Bip	768	64	16	3	4	3	W(6,2)
C4[12, 2]	12	24	DT	W	NB	48	4	1	2	3	3	R_6(5,4)
C4[13, 1]	13	26	DT	W	NB	52	4	1	0	4	2	C_13(1,5)
C4[14, 1]	14	28	DT	U	NB	(2^8)(7^1)	128	32	1	4	3	W(7,2)
C4[14, 2]	14	28	DT	W	Bip	336	24	6	0	4	3	BC_7(0124)
C4[15, 1]	15	30	DT	W	NB	60	4	1	2	4	3	C_15(1,4)
C4[15, 2]	15	30	DT	W	NB	120	8	2	0	3	3	Pr_5(1,1,2,2)
C4[16, 1]	16	32	DT	U	Bip	(2^12)	256	64	3	4	4	W(8,2)

C4[15, 2]	15	30	DT	W	NB	120	8	2	0	3	3	Pr_5(1,1,2,2)
C4[16, 1]	16	32	DT	U	Bip	(2^12)	256	64	3	4	4	W(8,2)
C4[16, 2]	16	32	DT	W	Bip	384	24	6	2	4	4	R_8(6,5)
C4[17, 1]	17	34	DT	W	NB	68	4	1	0	4	3	C_17(1,4)
C4[18, 1]	18	36	DT	U	NB	(2^10)(3^2)	512	128	1	4	4	W(9,2)
C4[18, 2]	18	36	DT	W	Bip	144	8	2	2	4	3	DW(6,3)
C4[20, 1]	20	40	DT	U	Bip	(2^12)(5^1)	(2^10)	256	3	4	5	W(10,2)
C4[20, 2]	20	40	DT	W	NB	320	16	4	1	4	4	R_10(7,6)
C4[20, 3]	20	40	DT	W	Bip	80	4	1	1	4	4	R_10(4,1)
C4[20, 4]	20	40	SS	U	Bip	(2^8)(3^1)(5^1)	384	96	0	4	4	SDD(K5)
C4[21, 1]	21	42	DT	W	NB	84	4	1	2	4	3	C_21(1,8)
C4[21, 2]	21	42	DT	W	NB	336	16	4	2	3	3	PS(3,7;2)
C4[22, 1]	22	44	DT	U	NB	(2^12)(11^1)	(2^11)	512	1	4	5	W(11,2)
C4[24, 1]	24	48	DT	U	Bip	(2^15)(3^1)	(2^12)	(2^10)	3	4	6	W(12,2)
C4[24, 2]	24	48	DT	W	Bip	96	4	1	3	4	4	C_24(1,5)
C4[24, 3]	24	48	DT	W	Bip	96	4	1	3	4	4	C_24(1,7)
C4[24, 4]	24	48	DT	W	Bip	768	32	8	2	4	4	R_12(8,7)
C4[24, 5]	24	48	DT	W	NB	96	4	1	2	3	4	R_12(11,4)
C4[24, 6]	24	48	DT	W	NB	96	4	1	2	5	3	R_12(5,10)
C4[24, 7]	24	48	SS	U	Bip	(2^10)(3^1)	256	64	0	4	4	SDD(Octahedron)
C4[25, 1]	25	50	DT	W	NB	100	4	1	0	4	3	C_25(1,7)
C4[25, 2]	25	50	DT	W	NB	200	8	2	1	4	4	{4,4}_5,0
C4[26, 1]	26	52	DT	U	NB	(2^14)(13^1)	(2^13)	(2^11)	1	4	6	W(13,2)
C4[26, 2]	26	52	DT	W	Bip	104	4	1	1	4	5	C_26(1,5)

Summary for C4[24, 5] = R_12(11,4)

[Home] [Table] [Glossary] [Families]

V	E	|AG|	vs	ds	Transitivity	Worthy?	Bipartite?	girth	diameter	#STO's
24	48	96	4	1	Dart-Transitive	Worthy	NB	3	4	2

Graph

Constructions

Related Graphs

Distance-Orbit Chart:

0	1	2	3	4
1	4	4^2	2,4^2	1

Lengths of Consistent Cycles

3	8	12

Cycle structures of semi-regular symmetries = $12^{2}, 8^{3}, 8^{3}$

 Graph forms for C4 [24, 5] = R_12(11,4)

On this page are computer-accessible forms for the graph C4[24, 5] = R_12(11,4).

(I) Following is a form readable by MAGMA:

g:=Graph<24|{ {2, 3}, {10, 11}, {8, 9}, {4, 5}, {6, 7}, {12, 13}, {1, 2}, {9, 10}, {5, 6}, {16, 20}, {17, 21}, {18, 22}, {19, 23}, {3, 4}, {11, 12}, {16, 24}, {1, 13}, {20, 24}, {2, 14}, {3, 15}, {1, 12}, {2, 15}, {1, 14}, {7, 8}, {3, 16}, {7, 20}, {11, 24}, {4, 16}, {7, 19}, {5, 17}, {6, 18}, {12, 24}, {4, 17}, {6, 19}, {5, 18}, {13, 21}, {14, 22}, {15, 23}, {8, 20}, {10, 22}, {9, 21}, {11, 23}, {13, 17}, {14, 18}, {15, 19}, {8, 21}, {10, 23}, {9, 22} }>;

(II) A more general form is to represent the graph as the orbit of {2, 3} under the group generated by the following permutations:

a: (2, 12)(3, 11)(4, 10)(5, 9)(6, 8)(13, 14)(15, 24)(16, 23)(17, 22)(18, 21)(19, 20)
b: (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12)(13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24)
c: (2, 13)(3, 17)(5, 16)(6, 20)(8, 19)(9, 23)(11, 22)(12, 14)(15, 21)(18, 24)

Summary for C4[24, 5] = R_12(11,4)

[Home] [Table] [Glossary] [Families]

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Constructions

Constructions for C4[24, 5] = R_12(11,4)

[Home] [Table] [Glossary] [Families]

On this page are all constructions for C4[24, 5]. See Glossary for some detail.

R_ 12(11, 4) = Pr_ 8(1, 1, 5, 5) = KE_ 6(2, 0, 1, 1, 2)

= UG(ATD[24, 7] = UG(ATD[24, 8] = L(F 16)

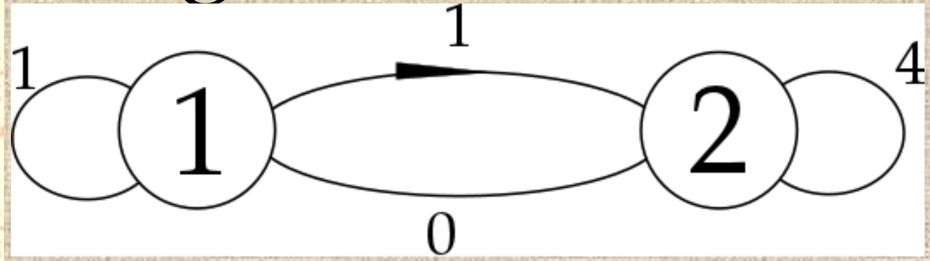
= MG(Rmap(24, 3) { 3, 8| 8}_ 12) = DG(Rmap(24, 3) { 3, 8| 8}_ 12) = DG(Rmap(24, 5) { 3, 12| 12}_ 8)

= AT[24, 3]

Diagrams

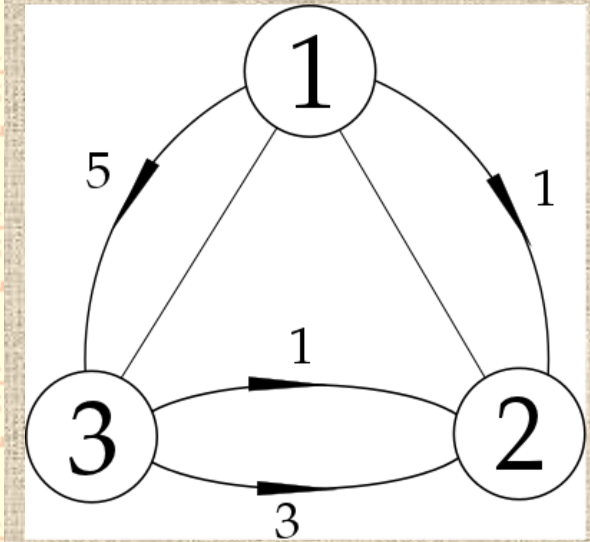
mod 12:

	1	2
1	1 11	0 1
2	0 11	4 8



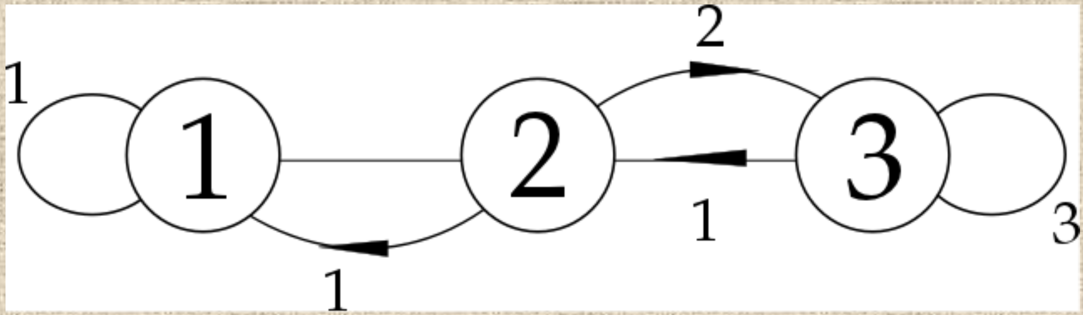
mod 8:

	1	2	3
1	-	0 1	0 5
2	0 7	-	5 7
3	0 3	1 3	-



mod 8:

	1	2	3
1	1 7	0 7	-
2	0 1	-	2 7
3	-	1 6	3 5



Summary for C4[24, 5] = R_12(11,4)

[Home] [Table] [Glossary] [Families]

V	E	|AG|	vs	ds	Transitivity	Worthy?	Bipartite?	girth	diameter	#STO's
24	48	96	4	1	Dart-Transitive	Worthy	NB	3	4	2

Graph

Constructions

Related Graphs

Distance-Orbit Chart:

0	1	2	3	4
1	4	4^2	2,4^2	1

Lengths of Consistent Cycles

3	8	12

Cycle structures of semi-regular symmetries = 12^{2}, 8^{3}, 8^{3}

Covers Down

 Graphs related to C4[24, 5] = R_12(11,4)

[Home] [Table] [Glossary] [Families]

On this page are all graphs related to C4[24, 5].

Graphs which this one covers

4-fold cover of C4[6, 1] = Octahedron

2-fold cover of C4[12, 2] = R_6(5,4)

Covers Up

Graphs which cover this one

2-fold covered by C4[48, 12] = KE_12(1,3,8,5,1)

3-fold covered by C4[72, 6] = R_36(11,28)

3-fold covered by C4[72, 13] = Pr_24(1,1,5,5)

4-fold covered by C4[96, 36] = UG(ATD[96, 11]

4-fold covered by C4[96, 39] = UG(ATD[96, 55]

4-fold covered by C4[96, 40] = UG(ATD[96, 61]

5-fold covered by C4[120, 11] = R_60(47,16)

5-fold covered by C4[120, 25] = Pr_40(1,33,37,29)

6-fold covered by C4[144, 25] = KE_36(1,3,20,17,1)

6-fold covered by C4[144, 40] = UG(ATD[144, 69]

6-fold covered by C4[144, 41] = UG(ATD[144, 72]

Aut-Orbitals

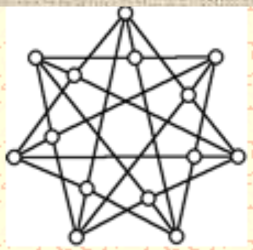
Aut-Orbital graphs of this one:

C4[8, 1] = K_4,4

C4[24, 5] = R_12(11,4)

C4[24, 6] = R_12(5,10)

Glossary



[Home] [Table] [Glossary] [Families]

GLOSSARY

GRAPH: In this census, the word *graph* means a simple graph, a colection V of things called "vertices" and a set E of unordered pairs from V, the edges of the graph.

SYMMETRY: A *symmetry*, also called an *automorphism*, of a graph is a permutation of its vertices which preserves edges. If Γ is a graph, the symmetries of Γ form a group under composition, called Aut(Γ).

Graphs in this Census: This Census concerns graphs of degree 4 (*tetravalent* graphs) which are *edge-transitive*, i.e., those for which, given edges e and e', there is σ in Aut(Γ) which sends e to e'. A *dart* (or *arc* or *directed edge*) is one of the two ordered pairs of vertices corresponding to an edge. If AG = Aut(Γ) is transitive on darts, we naturally call Γ *dart-transitive* (abbreviated DT); the word "symmetric" is often used with this meaning. If AG is transitive on edges and vertices, but not on darts, we say that Γ is *½-arc-transitive* or HT. If AG is transitive on edges but not on vertices, we say that Γ is *semi-symmetric*, or SS for short.

The headings in the Table and the spreadsheet version of the Census:

Tag: The tag "C4[20,5]" means this graph is number 5 in the list of graphs having 20 vertices.

Summary page

Constructions for C4[24, 5] = R_12(11,4)

[Home] [Table] [Glossary] [Families]

On this page are all constructions for C4[24, 5]. See Glossary for some detail.

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= UG(ATD[24, 7] = UG(ATD[24, 8] = L(F 16)

= MG(Rmap(24, 3) { 3, 8| 8}_ 12) = DG(Rmap(24, 3) { 3, 8| 8}_ 12) = DG(Rmap(24, 5) { 3, 12| 12}_ 8)

= AT[24, 3]

Propellor

DIAGRAMS

Propellor Graphs: The graph $Pr_n(a, b, c, d)$ is given by the diagram below.

This means that it has 3n vertices. They are A_i, B_i and C_i for numbers i mod n. There are 3 kinds of edges:

(1) Tip: $\{A_i, A_{i+a}\}$,$\{C_i, C_{i+d}\}$,

(2) Flat: $\{A_i, B_i\}$,$\{B_i, C_i\}$,

(3) Blade: $\{B_i, A_{i+b}\}$,$\{B_i, C_{i+c}\}$.

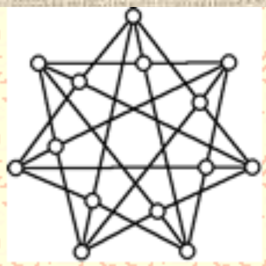
Those which are edge-transitive must, as a result, be dart-transitive. **Matthew Sterns** has found three families (one is finite) which are edge-transitive. He has recently proved that these include all edge-transitive Propellor graphs.

(a) (a, b, c, d) = (1, 2d, 2, d) for d satisfying $d^2 = \pm 1$.

(b) (a, b, c, d) = (1, b, b+4, 2b+3), for n divisible by 4, b = 1 (mod n), 8b+16 = 0 mod n.

(c) [n,a, b, c, d] = [5,1,1,2,2], [10,1,1,2,2], [10,1,4,3,2], [10,1,1,3,3], or [10, 2, 3, 1, 4].

Families Page



[Home] [Table] [Glossary] [Families]

A Census of edge-transitive tetravalent graphs
NewMini Census by Families

This page lists the tag and then the name of each graph resulting from any of the constructions listed in the [Glossary] .

*-AMC(2, 3 [0 1: 2 2]) = C4[18, 2] = DW(6,3)
*-AMC(2, 4 [0 1: 3 3]) = C4[16, 2] = R_8(6,5)
*-AMC(2, 5 [0 2: 3 0]) = C4[50, 3] = {4,4}_5,5
*-AMC(2, 7 [5 3: 6 2]) = C4[98, 2] = {4,4}_7,7
*-AMC(2, 8 [1 2: 2 7]) = C4[32, 4] = PX(4,3)
*-AMC(2, 8 [1 7: 0 7]) = C4[64, 5] = {4,4}_8,0
*-AMC(2, 9 [1 6: 6 4]) = C4[54, 2] = DW(18,3)
*-AMC(2, 12 [1 3: 0 7]) = C4[48, 5] = R_24(22,13)
*-AMC(2, 12 [1 4: 0 1]) = C4[36, 3] = DW(12,3)
*-AMC(2, 12 [1 11: 0 11]) = C4[144, 12] = {4,4}_12,0
*-AMC(3, 3 [0 1: 2 2]) = C4[27, 3] = AMC(3, 3 [0 1: 2 2])
*-AMC(3, 5 [0 1: 4 4]) = C4[75, 3] = AMC(3, 5 [0 1: 4 4])
*-AMC(3, 7 [0 1: 6 6]) = C4[147, 6] = AMC(3, 7 [0 1: 6 6])
*-AMC(3, 7 [0 4: 3 3]) = C4[147, 5] = MSZ(21,7,8,2)
*-AMC(3, 8 [5 5: 5 2]) = C4[48, 14] = AMC(3, 8 [5 5: 5 2])

Families Page

*-PX(30, 2) = C4[120, 8] = R_60(32,31)
*-PX(31, 2) = C4[124, 2] = R_62(33,32)
*-PX(32, 2) = C4[128, 2] = R_64(34,33)
*-PX(33, 2) = C4[132, 4] = R_66(35,34)
*-PX(34, 2) = C4[136, 4] = R_68(36,35)
*-PX(35, 2) = C4[140, 4] = R_70(37,36)
*-PX(36, 2) = C4[144, 4] = R_72(38,37)
*-PX(37, 2) = C4[148, 2] = R_74(39,38)
*-Pr_ 4(1, 1, 1, 1) = C4[12, 2] = R_6(5,4)
*-Pr_ 4(1, 2, 2, 1) = C4[12, 1] = W(6,2)
*-Pr_ 5(1, 1, 2, 2) = C4[15, 2] = Pr_5(1,1,2,2)
*-Pr_ 5(1, 4, 2, 2) = C4[15, 2] = Pr_5(1,1,2,2)
*-Pr_ 6(1, 2, 2, 1) = C4[18, 2] = DW(6,3)
*-Pr_ 8(1, 1, 5, 5) = C4[24, 5] = R_12(11,4)
*-Pr_ 8(1, 2, 2, 1) = C4[24, 3] = C_24(1,7)
*-Pr_ 8(1, 5, 1, 5) = C4[24, 6] = R_12(5,10)
*-Pr_ 8(1, 6, 2, 3) = C4[24, 2] = C_24(1,5)
*-Pr_ 10(1, 1, 2, 2) = C4[30, 4] = Pr_10(1,1,2,2)
*-Pr_ 10(1, 1, 3, 3) = C4[30, 6] = Pr_10(1,1,3,3)
*-Pr_ 10(1, 2, 2, 1) = C4[30, 2] = C_30(1,11)
*-Pr_ 10(1, 4, 3, 2) = C4[30, 5] = Pr_10(1,4,3,2)

The Recipes Paper

"Recipes for Edge-Transitive Tetravalent Graphs"

Contains:

- Recipes for all families of graphs in the Census
- Connections and overlaps between Families
- Theorems and lesser results
- A dozen open problems

Other Page