A census of cubic vertex-transitive graphs

Gabriel Verret (Primorska), P. Potočnik and P. Spiga

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Vertex-transitive graphs

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WLOG, we may assume connectedness.
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Using ad hoc methods, McKay and Royle (1996) obtained a list which is complete up to 94 vertices.

Using some new theoretical results and a few tricks, we constructed all cubic vertex-transitive graphs of order at most 1280.
Three natural cases

Let $\Gamma$ be a cubic $G$-vertex-transitive graph and let $m$ be the number of orbits of $G_{\Gamma(\nu)}$ (the permutation group induced by the action of a vertex-stabiliser $G_{\nu}$ in its action on the neighbourhood $\Gamma(\nu)$).
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By vertex-transitivity, $m$ is equal to the number of orbits of $G$ in its action on the arcs of $\Gamma$ (an arc is an ordered pair of adjacent vertices).
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We deal with each of these separately.
m = 1 (the arc-transitive case)

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$|G|$ grows at most linearly with $|V(\Gamma)|$ and the amalgams are known (the structure of $G_v$ and $G_{\{uv\}}$).
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To find all the graphs up to a certain order, it suffices to:

- Construct the amalgams (finitely generated amalgamated products of finite groups). There are 7 of these.
- Find all the normal subgroups up to a certain index (by using the `LowIndexNormalSubgroups` algorithm in Magma for example).

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Computationally infeasible.
A few tricks

Lemma

$G / G'$ is isomorphic to one of $\mathbb{Z}^3_2$, $\mathbb{Z}_2 \times \mathbb{Z}_r$, or $\mathbb{Z}_r$. 

Reduces drastically the number of groups we need to consider.

Example: 1090235 non-isomorphic groups of order 768, but only 4810 satisfy this Lemma.

Lemma

Let $G$ be a group and let $\phi \in \text{Aut}(G)$. Then $\text{Cay}(G, S) \cong \text{Cay}(G, S^\phi)$. 

Only need to consider connection sets up to conjugacy in $\text{Aut}(G)$.

These simple tricks are enough to make the $m=3$ case computationally feasible, except when $G$ has order 512 or 1024 (too many groups).
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2-groups

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Using this Lemma, we can construct $R_i$ by induction on $i$.

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Once we have constructed \( R_{512} \) and \( R_{1024} \), we apply to the groups in these classes the same procedure which we used for other orders.
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We define an auxiliary graph, which is 4-valent, $G$-arc-transitive and has half the order. We also get a $G$-arc-transitive cycle decomposition of this new graph.
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This construction is reversible, hence it suffices to find all 4-valent arc-transitive graphs and their arc-transitive cycle decompositions.

By a paper of Miklavec, Potočnik and Wilson, arc-transitive cycle decompositions of 4-valent graphs are well-understood, so it suffices to find all 4-valent arc-transitive graphs of order at most 640.
Because \( \Gamma \) admits an arc-transitive cycle-decomposition, we have \( G_{\Gamma(v)} \cong \mathbb{Z}_4, \mathbb{Z}_2^2 \) or \( D_4 \).
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We characterised the graphs for which $|G_v|$ is “very large” with respect to the order of the graph.
4-valent arc-transitive graphs

Theorem (PSV)

Let \((\Gamma, G)\) be locally-\(D_4\). Then one of the following holds:

- \(\Gamma \cong C(r, s)\),
- \((\Gamma, G)\) is one of 18 exceptions,
- \(|V\Gamma| \geq 2|G_v| \log_2(|G_v|/2)\). Moreover, the graphs for which equality occurs are determined.
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If $|V\Gamma| \leq 640$, then $|G_v| \leq 32$ or $\Gamma$ is “understood”.

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Side note: by combining our data with the census of small 2-arc-transitive 4-valent graphs (Potočnik), we get all 4-valent arc-transitive graphs of order at most 640.
Number of graphs of order up to $n$

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This seems to be part of a trend.
It is conjectured that almost all vertex-transitive graphs are Cayley (McKay and Praeger). It seems reasonable to conjecture that this is also true for any given valency $k \geq 3$. 

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We tested the graphs for various problems and conjectures (Hamiltonicity, degree/diameter, cages...)

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Asymptotic enumeration of $k$-valent vertex-transitive graphs (arc-transitive graphs, Cayley graphs, GRRs).

Question
Are almost all $k$-valent vertex-transitive graphs Cayley? GRRs?

Challenge
Census of $4$-valent vertex-transitive graphs of order up to 200? 300?

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