

PhD opportunities in Applied Dynamical Systems

Applications are invited for two fully-funded PhD studentships in Applied Dynamical Systems in the Department of Mathematics at the University of Auckland, New Zealand. The students will be supervised by Associate Professors Claire Postlethwaite and Vivien Kirk.

Project descriptions

Heteroclinic networks [1] are special types of solutions to dynamical systems that can be found in mathematical models describing a diverse range of physical systems, from fluid dynamics to population models and cognitive functions. It is well-known that noise can modify the behavior of dynamical systems, but the effects of noise on heteroclinic networks can be counter-intuitive and are not well understood. One project will consider noise-induced memory in heteroclinic networks, and the second will use heteroclinic network methods to design a model for cognitive processes. Further details of the projects can be found below.

Other information

The successful candidates for both projects will hold, or expect to complete soon, a Bachelor's or Master's degree in Pure or Applied Mathematics, or similar. They should have a strong background in dynamical systems and computational mathematics. Some knowledge of statistics and probability will be an advantage.

Both students will work within the dynamical systems group at the University of Auckland. The group currently has seven research-active staff, five postdoctoral fellows, and around ten PhD students. Both projects will be in collaboration with Professor Peter Ashwin (University of Exeter, UK), and the students will have the opportunity to visit Professor Ashwin in Exeter during their studies, as well as attend international conferences to present their work.

Informal enquiries are welcome by email, to Claire Postlethwaite (c.postlethwaite@auckland.ac.nz)

Stipend is NZ\$27,500 pa (tax free) for three years plus tuition fees. Start date is flexible but would preferably be between March and November 2018. Interested candidates should send an email expressing their interest, along with a CV, academic record, and list of three potential referees to Associate Professor Claire Postlethwaite at c.postlethwaite@auckland.ac.nz. Applications will be considered until the position is filled; applications received by Dec 15th, 2017, will receive full consideration.

1. Noise-induced memory in heteroclinic networks

Not surprisingly, noise affects the residence times and the transition probabilities of switching between the states in a heteroclinic network. However, a particularly surprising phenomena is the appearance of long-time correlations, or memory, in the sequence of transitions between states when microscopic

noise is added to the system. The memory can be understood as a consequence of *lift-off* [2], which causes the distribution of trajectories near the network to be asymmetric. The research aim for this project will be to answer several questions regarding the properties of noise-induced memory in heteroclinic networks. Specifically: can lift-off affect residence times at states in a network? Are there limits to the length of time for which lift-off induced memory can remain? Can the sequence of states be modelled by an n th order Markov Chain? Is there a limit to the amount of information that can be encoded in the distribution of the coordinates of the trajectory? What are the properties of lift-off if the states in the network are more complicated objects than equilibria? These problems will be attacked with a combination of analytic and computational methods.

2. Modelling cognitive functions using heteroclinic networks

Heteroclinic networks are particularly suited to the modelling of cognitive functions due to the sequential and intermittent nature of the dynamics as the trajectory explores the network. Recent work by Postlethwaite and Ashwin [3] develops new techniques for designing deterministic systems containing heteroclinic networks of any specified topology. The proposed research for this project is to use these techniques to design heteroclinic networks to model cognitive functions. One possible such task is *cognitive task switching* [4]. Switching between cognitive tasks is known to cause an increase in both reaction times and error rates even when a subject is practised in all the different tasks. One of the biggest challenges with conceptual modelling such as this is designing a model from which you get more out than you put in. That is, we must ensure the model doesn't merely reproduce the phenomena of interest, but is able to provide us with insight into why the phenomena occurs, or is able to make predictions about new phenomena which can be tested in experiments. The interpretation of the analysis of the network will be done in collaboration with experimental psychologists, to make predictions about questions such as: How does the switch cost vary as the relative ease or difficulty of the two tasks are altered? Can we predict the effect on the switch cost of having more than two tasks? Are the two components of the switch cost (delay time and accuracy rate) always related to each other or can they be adjusted independently? Answers to such questions will guide the design of future experiments to test our predictions.

An interest in cognitive science will be an advantage for this project, but no prior knowledge is required.

References

- [1] Kirk, V., Silber, M. (1994) A competition between heteroclinic cycles. *Nonlinearity*, 7 1605–1621.
- [2] Armbruster, D., Kirk, V., Stone, E. (2003) Noisy heteroclinic networks. *Chaos*, 13(1), 71–79.
- [3] Ashwin, P., Postlethwaite, C.M. (2013) On designing heteroclinic networks from graphs. *Physica D*, 265, 26–39.
- [4] Monsell, S. (2003) Task switching. *TRENDS in Cognitive Sciences*, 7(3), 134–140.