ON GENERALIZED CLOSED SETS

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Abstract

In this paper, we study generalized closed sets in a topological space. The question of when some classes of generalized closed sets coincide is considered. Some lower separation axioms weaker than T_1 are investigated. Characterizations of extremally disconnected spaces and sg-submaximal spaces are given by using various kinds of generalized closed sets.

1 Introduction

In a topological space, closed sets are fundamental objects, which are as important as open sets. For instance, we can form the topology on a set by using either axioms of closed sets or the Kuratowski closure axioms. In 1970, Levine [16] initiated the study of generalized closed sets. They have been studied intensively by some topologists in recent years. This is because generalized closed sets are not only natural generalizations of closed sets. More importantly, they also suggest some new properties of topological spaces. Most of these new properties are separation axioms weaker than T_1, some of which have been found to be useful in computer science and digital topology. For example, the well-known digital line is $T_{3/4}$ but not T_1. Furthermore, the study of generalized closed sets also provides some new characterizations of peculiar spaces. One such class of spaces is the class of extremally disconnected spaces. Others are defined by variations of the property of submaximality.

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A subset S of a topological space X is called *α-open* [resp. *semi-open, preopen, semi-preopen*] if $S \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}S))$ [resp. $S \subseteq \mathrm{cl}(\mathrm{int}S)$, $S \subseteq \mathrm{int}(\mathrm{cl}S)$, $S \subseteq \mathrm{cl}(\mathrm{int}(\mathrm{cl}S))$]. Moreover, S is said to be *α-closed* [resp. *semi-closed, preclosed, semi-preclosed*] if $X \setminus S$ is α-open [resp. semi-open, preopen, semi-preopen] or, equivalently, if $\mathrm{cl}(\mathrm{int}(\mathrm{cl}S)) \subseteq S$ [resp. $\mathrm{int}(\mathrm{cl}S) \subseteq S$, $\mathrm{cl}(\mathrm{int}S) \subseteq S$, $\mathrm{int}(\mathrm{cl}(\mathrm{int}S)) \subseteq S$]. The *$\alpha$-closure* [resp. *semi-closure, preclosure, semi-preclosure*] of $S \subseteq X$ is the smallest α-closed [resp. semi-closed, preclosed, semi-preclosed] set containing S. It is well-known that $\alpha\text{-cl}S = S \bigcup \mathrm{cl}(\mathrm{int}(\mathrm{cl}S))$ and $\mathrm{scl}S = S \bigcup \mathrm{int}(\mathrm{cl}S)$, $\mathrm{pcl}S = S \bigcup \mathrm{cl}(\mathrm{int}S)$ and $\mathrm{spcl}S = S \bigcup \mathrm{int}(\mathrm{cl}(\mathrm{int}S))$. The *$\alpha$-interior* of $S \subseteq X$ is the largest α-open set contained in S, and we have $\alpha\text{-int}S = S \bigcap \mathrm{int}(\mathrm{cl}(\mathrm{int}S))$. It is worth mentioning that the collection $\alpha(X)$ of all α-open subsets of X is a topology on X [21] which is finer than the original one, and that a subset S of X is α-open if and only if S is semi-open and preopen [23].

Definition 1. Let X be a topological space. A subset A of X is called

(1) *generalized closed* (briefly, *g-closed*) [16] if $\mathrm{cl}A \subseteq U$, whenever $A \subseteq U$ and U is open;

(2) *semi-generalized closed* (briefly, *sg-closed*) [2], if $\mathrm{scl}A \subseteq U$ whenever $A \subseteq U$ and U is semi-open;

(3) *generalized semiclosed* (briefly, *gs-closed*) [1] if $\mathrm{scl}A \subseteq U$ whenever $A \subseteq U$ and U is open;

(4) *generalized α-closed* (briefly, *$g\alpha$-closed*) [18], if $\alpha\text{-cl}A \subseteq U$ whenever $A \subseteq U$ and U is α-open, or equivalently, if A is g-closed with respect to $\alpha(X)$;

(5) *α-generalized closed* (briefly, *αg-closed*) [19] if $\alpha\text{-cl}A \subseteq U$ whenever $A \subseteq U$ and U is open;

(6) *generalized semi-preclosed* (briefly, *gsp-closed*) [6] if $\mathrm{spcl}A \subseteq U$ whenever $A \subseteq U$ and U is open;

(7) *regular generalized closed* (briefly, *r-g-closed*) [22] if $\mathrm{cl}A \subseteq U$ whenever $A \subseteq U$ and U is regular open.

In addition to Definition 1 above, a subset A of X is *g-open* [16] [*sg-open* [2]] if $X \setminus A$ is g-closed [sg-closed]. Other generalized open sets can be defined similarly. Recall that a space X is said to be *submaximal* if every dense subset of X is open. As variations of submaximality, we obtain the notions of α-submaximality, g-submaximality and sg-submaximality. Recall that a space X is *α-submaximal* [resp. *g-submaximal*, *sg-submaximal*] if every dense subset is α-open (resp. g-open, sg-open). In [12], Ganster studied the class of

α-submaximal spaces. Obviously, every submaximal space is g-submaximal, and every g-submaximal space is sg-submaximal. Note that any indiscrete space with at least two points is g-submaximal but not submaximal, and an sg-submaximal space which is not g-submaximal was given in [4].

In [7], Dontchev summarized the fundamental relationships between various types of generalized closed sets in the following diagram. Note that none of the implications in the diagram below is reversible, in general.

$$\begin{tikzcd}
\text{closed set} \arrow[r] \arrow[d] & \text{g-closed set} \arrow[r] \arrow[dr] \arrow[drr] & \alpha\text{g-closed set} \arrow[d] \arrow[ddr] & \\
\alpha\text{-closed set} \arrow[r] \arrow[d] & \text{g}\alpha\text{-closed set} \arrow[r] \arrow[ur] \arrow[ddr] & \text{gs-closed set} \arrow[dr] & \text{r-g-closed set} \\
\text{semi-closed set} \arrow[r] \arrow[d] & \text{sg-closed set} \arrow[ur] \arrow[rr] \arrow[dl] & & \text{gsp-closed set} \\
\text{semi-preclosed set} \arrow[urrr] & & \text{preclosed set} \arrow[ll] \arrow[ur] &
\end{tikzcd}$$

Concerning converses of some implications in the above diagram, Dontchev [7] posed two questions asking in which spaces every semi-preclosed subset is sg-closed, and in which spaces every preclosed subset is $g\alpha$-closed. These two questions and other possible converses of some implications in the diagram were considered by Cao, Ganster and Reilly in [3, 4]. This leads to some new characterizations of T_{gs}-spaces, extremally disconnected spaces and sg-submaximal spaces. The main purpose of this paper is to summarise recent work in this direction, and to present some new results.

Throughout this paper, no separation axioms are assumed unless it is stated explicitly.

2 Dontchev's questions

In a recent paper [7], Dontchev posed the following two open questions regarding generalized closed sets:

Question 2.1. *Characterize the spaces in which every semi-preclosed set is sg-closed.*

Question 2.2. *Characterize the spaces in which every preclosed set is $g\alpha$-closed.*

To answer these questions, we need some notation and lemmas. Let S be a subset of a space X. A *resolution* of S is a pair $< E_1, E_2 >$ of disjoint dense subsets of S. Furthermore, S is said to be *resolvable* if it possesses a resolution, otherwise S is called *irresolvable*. In addition, S is called *strongly irresolvable*, if every open subspace of S is irresolvable. Observe that if $< E_1, E_2 >$ is a resolution of S then E_1 and E_2 are codense in X, i.e. have empty interior. Note that every submaximal space is hereditarily irresolvable.

Lemma 2.3. *[14, 12] Every space X has a unique decomposition $X = F \bigcup G$, where F is closed and resolvable and G is open and hereditarily irresolvable.*

In this paper, $X = F \bigcup G$, where F and G are defined in Lemma 2.3, is called the *Hewitt decomposition*.

Lemma 2.4. *[15] Let X be a space. Then every singleton of X is either nowhere dense or preopen.*

For a space X, define $X_1 = \{x \in X : \{x\}$ is nowhere dense $\}$, and $X_2 = \{x \in X : \{x\}$ is preopen $\}$. Then $X = X_1 \bigcup X_2$ is a decomposition of X, which is called the *Janković-Reilly decomposition*. In addition, a space X is called *locally indiscrete* if every open subset is closed. The following theorem was proved in [3] in answering Question 2.1 and Question 2.2.

Theorem 2.5. *[3] For a space X the following are equivalent:*
(1) every semi-preclosed subset of X is sg-closed,
(2) $X_1 \bigcap \mathrm{scl}A \subseteq \mathrm{spcl}A$ for each $A \subseteq X$,
(3) $X_1 \subseteq \mathrm{int}(\mathrm{cl}G)$,
(4) $X \approx Y \oplus Z$, where Y is locally indiscrete and Z is strongly irresolvable,
(5) every preclosed subset of X is $g\alpha$-closed,
(6) X is g-submaximal with respect to $\alpha(X)$.

3 Lower separation axioms

A space X is $T_{1/2}$ [16] if every g-closed subset of X is closed. It was shown in [11] that a space X is $T_{1/2}$ if and only if singletons of X are either open or closed. Other interesting results of this class of spaces can be found in [8] and [11].

Theorem 3.1. *[5] Every submaximal space is $T_{1/2}$.*

Clearly, in a topological space X, every sg-closed subset of X is gs-closed. In [20], a new class of spaces called T_{gs}-space was introduced. Recall that a space X is T_{gs} if every gs-closed subset of X is sg-closed. One may ask how T_{gs}-spaces are related to $T_{1/2}$-spaces.

Theorem 3.2. *[4, 8] For a space X, the following statements are equivalent:*

(1) X is a T_{gs}-space,

(2) every nowhere dense subset of X is a union of closed subsets of X (i.e. X is T_1^* [13]),

(3) every generalized semi-preclosed subset of X is semi-preclosed (i.e. X is semi-pre-$T_{1/2}$ [6]),

(4) every singleton of X is either preopen or closed.

Corollary 3.3. *Every $T_{1/2}$-space is T_{gs}.*

A space X is *semi-T_1* [17] if every singleton is semi-closed and X is *semi-$T_{1/2}$* [2] if every singleton is either semiclosed or semiopen. Let $s(X)$ be the semi-regularization of a space X. The closure of a subset A of X with respect to $s(X)$ will be denoted by δ-cl$A$. A subset A of X is called *δ-generalized closed* if δ-cl$A \subseteq U$ when $A \subseteq U$ and U is open in X. Moreover, X is called a *$T_{3/4}$-space* [8] if every δ-generalized closed subset of X is closed in $s(X)$. The well-known digital line or the Khalimsky line is a $T_{3/4}$-space which fails to be T_1.

Theorem 3.4. *For any space X,*

(1) [8] $T_{3/4} = T_{gs}$ + semi-T_1.

(2) [20] $T_{1/2} = T_{gs}$ + semi-$T_{1/2}$.

The above results highlight the connections of the T_{gs} property to other lower separation axioms. However, not every T_{gs}-space is T_0. For example, a three point space $X = \{a, b, c\}$ in which the only proper open subset is $\{a, b\}$, is a T_{gs}-space, but not a T_0-space. To obtain more characterizations of T_{gs}-spaces, we need the following lemma.

Lemma 3.5. *A subset A of a space X is $g\alpha$-closed if and only if $X_1 \bigcap \alpha$-cl$A \subseteq A$.*

Proof. Suppose that A is $g\alpha$-closed, and let $x \in X_1 \bigcap \alpha\text{-cl}A$. If $x \notin A$, then $X \setminus \{x\}$ is an α-open set containing A and so $\alpha\text{-cl}A \subseteq X \setminus \{x\}$. This is impossible.

Conversely, suppose that we have $X_1 \bigcap \alpha\text{-cl}A \subseteq A$. Let U be α-open and contain A. Let $x \in \alpha\text{-cl}A$. If $x \in X_1$, then $x \in X_1 \bigcap \alpha\text{-scl}A \subseteq U$. If $x \in X_2$. Assume that $x \notin U$. Then $X \setminus U$ is α-closed and contains x. Thus,

$$\alpha\text{-cl}(\{x\}) = \{x\} \bigcup \text{cl}(\text{int}(\text{cl}(\{x\}))) \subseteq X \setminus U.$$

Since $\{x\}$ is preopen, $\{x\} \subseteq \text{cl}(\text{int}(\text{cl}\{x\}))$. It follows that $\text{int}(\text{cl}(\{x\})) \bigcap A \neq \emptyset$. Choose a point $y \in \text{int}(\text{cl}(\{x\})) \bigcap A$. Then $y \in A \bigcap (X \setminus U) \subseteq U \bigcap (X \setminus U)$. This leads to a contradiction. □

Theorem 3.6. *A space X is T_{gs} if and only if every αg-closed subset of X is $g\alpha$-closed.*

Proof. Suppose that X is T_{gs}. Let A be αg-closed and let $x \in X_1 \bigcap \alpha\text{-cl}A$. Then $\{x\}$ is closed. Assume that $x \notin A$, i.e. $A \subseteq X \setminus \{x\}$. Since A is αg-closed, $X \setminus \{x\}$ is open, then $x \in \alpha\text{-cl}A \subseteq X \setminus \{x\}$ which is a contradiction. Therefore, $X_1 \bigcap \alpha\text{-cl}A \subseteq A$. By Lemma 2.3, A is $g\alpha$-closed.

Conversely, assume that every αg-closed subset of X is $g\alpha$-closed. Let $x \in X_1$ and suppose that $\{x\}$ is not closed. Then $X \setminus \{x\}$ is an αg-closed, dense subset, thus $g\alpha$-closed. It follows from Lemma 2.3 that

$$X_1 \bigcap \alpha\text{-cl}(X \setminus \{x\}) = X_1 \bigcap X = X_1 \subseteq X \setminus \{x\}.$$

Therefore, we have $x \in X \setminus \{x\}$, which is a contradiction. □

Remark 3.7. A space whose αg-closed subsets are $g\alpha$-closed may be called a $T_{\alpha g}$*-space.* Theorem 3.6 shows that the class of $T_{\alpha g}$-spaces is precisely the class of T_{gs}-spaces.

Proposition 3.8. *A space X is both T_{gs} and extremally disconnected if and only if every gs-closed subset of X is preclosed.*

Proof. Suppose that X is both T_{gs} and extremally disconnected. Then every gs-closed subset is sg-closed. By Theorem 2.3 (3) of [4], every sg-closed subset is preclosed for an extremally disconnected space. Therefore, every gs-closed subset is preclosed.

Now assume that every gs-closed subset is preclosed. For each point $x \in X$, suppose that $\{x\}$ is not closed, then $X \setminus \{x\}$ is not open, so it is

gs-closed. By assumption, $X \setminus \{x\}$ is preclosed. Hence, $\{x\}$ is preopen. By Theorem 3.2, X is a T_{gs}-space. Moreover, since every sg-closed subset is preclosed, again by Theorem 2.3(3) of [4], X is extremally disconnected. $\square$

Proposition 3.9. *In a T_{gs}-space X, every g-closed subset is α-closed.*

Proof. Let A be g-closed. Suppose that there is a point $x \in \mathrm{cl}(\mathrm{int}(\mathrm{cl}A)) \setminus A$. If $\{x\}$ is preopen, then $\mathrm{int}(\mathrm{cl}(\{x\})) \bigcap \mathrm{int}(\mathrm{cl}A) \neq \emptyset$. So, $x \in \mathrm{int}(\mathrm{cl}A)$. Since $A \subseteq X \setminus \{x\}$, we have $\mathrm{int}(\mathrm{cl}A) \subseteq X \setminus \{x\}$. This is impossible. Otherwise, by Theorem 3.2, $\{x\}$ is closed. Then $x \in \mathrm{cl}(\mathrm{int}(\mathrm{cl}A)) \subseteq \mathrm{cl}A \subseteq X \setminus \{x\}$, which is a contradiction. $\square$

Question 3.10. *Characterize spaces whose α-closed subsets are g-closed.*

Remark 3.11. Certainly, in a nodec space X (i.e. every nowhere dense subset of X is closed [10]), every α-closed subset of X is g-closed. It is easy to show that spaces whose α-closed subsets are g-closed are precisely those spaces whose nowhere dense subsets are g-closed. Hence, Question 3.10 is equivalent to the following question: Is there a space whose nowhere dense subsets are g-closed but which has a non-closed nowhere dense subset? Such a space must be neither nodec nor $T_{1/2}$.

4 Characterizations of peculiar spaces

Refering to the diagram in Section 1, we now consider possible converses of some implications in that diagram thereby obtaining some more characterizations of some classes of peculiar spaces. One of them is the class of extremally disconnected spaces. The following theorem was proved in [4].

Theorem 4.1. *[4] For a space X, the following statements are equivalent:*
(1) (X, τ) is extremally disconnected,
(2) $\mathrm{scl}(A \bigcup B) = \mathrm{scl}A \bigcup \mathrm{scl}B$ for all $A, B \subseteq X$,
(3) the union of two semi-closed subsets of X is semi-closed,
(4) the union of two sg-closed subsets of X is sg-closed,
(5) every semi-preclosed subset of X is preclosed,
(6) every sg-closed subset of X is preclosed,
(7) every semi-closed subset of X is preclosed,
(8) every semi-closed subset of X is α-closed,
(9) every semi-closed subset of X is $g\alpha$-closed.

Next we present one more characterization of extremal disconnectedness using generalized closed subsets.

Theorem 4.2. *A space X is extremally disconnected if and only if every gs-closed subset of X is αg-closed.*

Proof. Suppose that X is extremally disconnected. Let A be gs-closed and let U be an open set containing A. Then $\mathrm{scl}A \subseteq U$, i.e. $\mathrm{int}(\mathrm{cl}A) \subseteq U$. Since $\mathrm{int}(\mathrm{cl}A)$ is closed, we have

$$\alpha\text{-}\mathrm{cl}A = A \bigcup \mathrm{cl}(\mathrm{int}(\mathrm{cl}A)) \subseteq A \bigcup \mathrm{int}(\mathrm{cl}A) \subseteq U.$$

Hence A is αg-closed.

Conversely, assume that every gs-closed subset of X is αg-closed. To show that X is extremally disconnected, let A be regular open. Since A is gs-closed, it is αg-closed. It follows that $\mathrm{cl}A = \mathrm{cl}(\mathrm{int}(\mathrm{cl}A)) = \alpha\text{-}\mathrm{cl}A \subseteq A$. Therefore A is closed. □

Next we study the property of sg-submaximality by using generalized sets. We first give some elementary characterizations of sg-submaximal spaces in the following theorem. Since its proof is straightforward, we omit it.

Theorem 4.3. *For a space X, the following are equivalent:*

(1) X is sg-submaximal,

(2) every subset of X is an intersection of a closed subset and an sg-open subset of X,

(3) every subset of X is a union of an open subset and an sg-closed subset of X,

(4) every subset A of X for which $\mathrm{int}A = \emptyset$, is sg-closed,

(5) $\mathrm{cl}A \setminus A$ is sg-closed for every subset A of X.

The following result about sg-submaximality was obtained in [4].

Theorem 4.4. *[4] For a space X, the following statements are equivalent:*

(1) $X_1 \subseteq \mathrm{cl}G$,

(2) every preclosed subset of X is sg-closed,

(3) X is sg-submaximal,

(4) X is sg-submaximal with respect to $\alpha(X)$.

Next we shall improve the equivalence of (2) and (3) in Theorem 4.4 and provide a new characterization of sg-submaximal spaces.

Theorem 4.5. *A space X is sg-submaximal if and only if every preclosed subset of X is gs-closed.*

Proof. The necessity is trivial by Theorem 4.4 (2). For the sufficiency, suppose that every preclosed subset is gs-closed. Let $X = F \bigcup G$ be the Hewitt decomposition of X, and let $< E_1, E_2 >$ be a resolution of int$F$.

We claim that every open set $V \subseteq \text{int}F$ is regular open. In fact, $V \bigcap E_1$ is codense and contained in V. Since codense sets are preclosed, by assumption, they are gs-closed. Thus, $\text{int}(\text{cl}(V \bigcap E_1)) \subseteq V$. On the other hand, E_1 is dense in int$F$, hence we have $\text{int}(\text{cl}(V \bigcap E_1)) = \text{int}(\text{cl}V)$. It follows that $V = \text{int}(\text{cl}V)$.

Now, let $x \in \text{int}F$. Let $V = \text{int}F \bigcap (X \setminus \text{cl}(\{x\}))$. Suppose that $\{x\}$ is nowhere dense. Then $X \setminus \{x\}$ is dense and

$$\text{int}(\text{cl}V) = \text{int}(\text{cl}(\text{int}F)) = \text{int}F.$$

By our claim, $\text{int}F = V$. Hence, $\text{int}F \subseteq X \setminus \{x\}$. This is a contradiction. Therefore, $\{x\}$ is preopen. We have proved that $\text{int}F \subseteq X_2$. However, this is equivalent to $X_1 \subseteq \text{cl}G$. By Theorem 4.4, X is sg-submaximal. ☐

Remark 4.6. One may define a space X to be *gs-submaximal* if each dense subset of X is gs-open. Similarly to the proof of Theorem 4.4, it is easily checked that a space X is gs-submaximal if and only if each preclosed subset of X is gs-closed. In the light of Theorem 4.5, the notion of gs-submaximality coincides with that of sg-submaximality.

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