Charles Dodgson: Mathematician

Charles Dodgson, who died just over a hundred years ago, is famous throughout the world as Lewis Carroll, the author of delightful children's stories. Rather than these innocent, but at the same time deep, fantasies about a little girl called Alice, we will discuss here some of his mathematical work.

In "A New Theory of Parallels", Dodgson attempted to prove a conjecture dating from the time of Euclid. Using well-accepting geometric arguments, it can be proved that if one is given a (straight) line in the plane and a point not on this line, then there exists at least one line through the given point parallel to the given line. The open question was of course whether there could be more than one such line. The belief that the line was unique, the "Euclidean Parallel Axiom", was almost universally held and failed attempts to prove it have led to many interesting geometric constructions. It is now known, and it was actually known in Dodgson's lifetime, that this can never be successful. In fact geometrical systems can exist with either one parallel through the point or more than one and in each of the two types geometry, it is possible to build a model of the other.

Not being part of the culture that looked at these remarkable discoveries, Dodgson publish a "proof" of the Euclidean Parallel Axioms. His idea of axioms as "self-evident truths" is at variance with modern mathematical thought. Today, we regard axioms as statements we can choose quite arbitrarily; what matters is the consequences of these axioms. The view of Dodgson's day was that geometrical and other fundamental truths are eternal and unshakeable and all we have to do is understand how to deduce them from axioms which we can take for granted as being obviously true. Dodgson's starting point was the belief that if you fit a square into a circular disc with each corner of the square is less than the area of the square. This seemed obvious to Dodgson but it is not true for very large circles and inscribed regular tetragons in non-Euclidean geometry. In Fact the area of circles is unbounded, just as for Euclidean geometry, whereas the area of convex polygons with a fixed number of edges is bounded.

We can learn more about Dodgson the man as well as Dodgson the mathematician by reading one of his works of popular mathematics known as the "Pillow Problems". These were designed to be solved with the head on the pillow as a remedy against thinking impious thoughts before one went to sleep. "There are sceptical thoughts, which seem to uproot the firmest faith; there are blasphemous thoughts, which dart unbidden into the most reverent souls; there are unholy thoughts, which torture, with their hateful presence, the fancy which would fain be pure." A lot can be learned about the mathematics of his day, especially about the mathematics that would have been known to Dodgson and his colleagues in Oxford, from the questions and answers in this little book. For example, many questions could not be precisely expressed in the framework that he used, because they involved ideas of limiting processes which were poorly understood in his group of mathematicians. It is still interesting to attempt these questions, and at the same time to ask oneself how the questions should be properly formulated from a modern standpoint.

Although some of the pillow problems are interesting exercises, many of them are standard or trivial mathematics or are absurd from a modern viewpoint. Furthermore, the author does not seem even to have known much of the mathematics that was current even in his own time. In presenting a sample from these problems I am deliberately avoiding the many that are challenging and still interesting in favour of those that have an eccentricity about them. I leave it to the reader to seek out the former and to see how many he or she can solve in the head before falling asleep. The Dover reprint of the book is in the catalogue of the Amazon Bookshop with ISBN number 0486204936 and a (20% discounted) price of $US6.36. They say, optimistically, that it has a reading age of 4 to 8. The numbered questions are Dodgson's. The commentaries are mine.

29. Prove the sum of 2 different squares, multiplied by the sum of 2 different squares, gives the sum of squares in 2 different ways. *The identity $(x^2+y^2)(u^2+v^2) = (xu \mp yv)^2 + (xv \pm yu)^2$ is well-known and can be written using complex numbers in the form $|z|^2|w|^2 = |zw|^2 = |z\bar{w}|^2$. It is used in the proof of the characterisation of positive integers which can be written as the sum of two squares.*

44. If a, b be two numbers prime to each other, a value may be found for n which will make $(a^n - 1)$ a multiple of b. *The famous theorems of Fermat and Euler seem to have been unknown to Dodgson who gives an argument for the case $a = 10$ based on the assertion that $1/b$ as a decimal eventually recurs. Other choices for a are then included in the hand-wave.*

45. If an infinite number of rods be broken: find the chance that at least one of them is broken in the middle. *The numerical answer is given as the number we now write as $1 - exp(-1)$ and the reason: divide the rod into $n+1$ parts where n is odd and assume that the n points of division are the only places where the rod will break, and that these are "equally frangible". Somehow n becomes also the number of rods and the answer is found to be $1 - (1-0)^{\frac{1}{0}}$.*

72. A bag contains 2 counters, as to which nothing is known except that each is either black or white. Ascertain their colours without taking them out of the bag. *The answer is given as one black and one white. Why? If there were two black counters and one white counter in the bag then the chance of drawing out a black counter would be $\frac{2}{3}$. If a black counter is added to the given bag, which originally had BB (probability $\frac{1}{4}$) or BW (probability $\frac{1}{2}$) or WW (probability $\frac{1}{4}$) it is deduced that the chance of drawing out a black counter would be this same value $\frac{2}{3}$.*

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