# Apology Number 15: "Lessons in Geometry"

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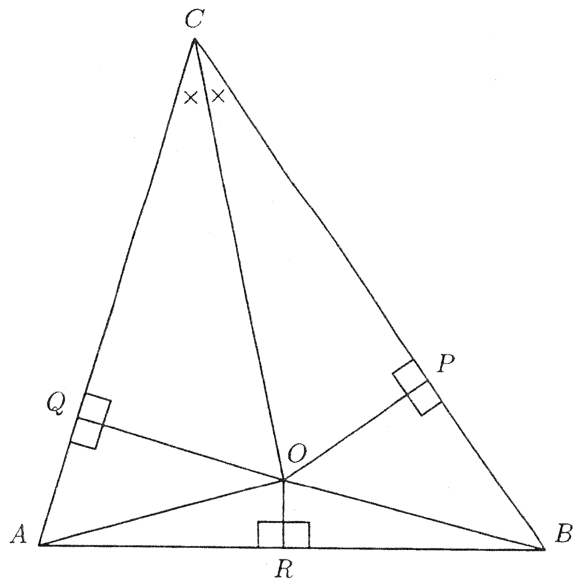
If Charles Dickens had visited a fifth form boys' geometry class at my old high school in 1949, and tried to use this as a model for a classroom scene in Nicholas Nickleby, he would never have got it published, because it would have been too extreme to have been believable.

The standard lesson consisted of three parts. In part one the boys were told to write out the proof of a "Theorem" they were supposed to have learnt by rote. In part two the boys were told to fill in their time looking at something or other in the textbook while the teacher looked through their written efforts. The 40 or so attempts were divided into two approximately equal piles on a pass-fail basis. In part three the boys whose work was deemed to be unsatisfactory were given two cuts with the cane on the bottom. There was then just sufficient time for the teacher to announce the number of the theorem which would be handled the same way in the next lesson.

I can't really judge objectively if being part of that class had any positive role in my own development as a mathematician. I never failed to get my theorem right and certainly I feared punishment, but geometry was one thing I was lucky enough to be reasonably good at. But if there really is such a thing as Mathematics phobia, it must have become quite common amongst members of this class.

Easy though it is to find examples of how geometry should not be taught, it is difficult to say how to teach it well. It is not even obvious what the aims should be and how competing aims should be balanced. Geometry is so natural to everybody with eyes to see and hands to touch, that we start our education with a wealth of prior knowledge. One aim might be to add to this knowledge by deducing consequences of what we already understand. Another aim might be to try to separate out intuitive but misleading ideas from what we know from our life experiences. Along with this comes an appreciation for logical thought processes and a mistrust of shoddy reasoning.

One of the great influences on my own life was Professor Henry George Forder. Looking back, I don't thank him for much of the technical knowledge of mathematics that I somehow picked up, but I do thank him for what I think I learned about the essential nature of mathematics. Because Geometry was his specialty, Professor Forder usually taught the single lecture each week in this topic to what used to be Pure Mathematics I. Because of the way things were done in those days, I skipped over the first year course and never saw him in action as a Pure Mathematics I Geometry lecturer. But I heard about some of what went on from other students. As a great personality, as well as a somewhat eccentric teacher, his fame was unequalled.




I have been told that he entered the lecture room on one April Fool's day in the 50s and saw on the blackboard some clumsy humour at his expense. He then dictated a theorem, not untypically, about a triangle $ABC$. The line $CO$ bisects the angle $ACB$ and meets the line $RO$ drawn at right angles to $AB$ through the centre of this line. The other lines shown in the diagram are joined up with the lines $OQ$ and $OP$ orthogonal to $AC$ and $BC$ respectively. A number of pairs of triangles are shown to be congruent using well-accepted rules about such pairs of triangles.

(1) Triangles $CQO$ and $CPO$ are congruent and consequently $CQ = CP$ and $QO = PO$.

(2) Triangles $ARO$ and $BRO$ are congruent and consequently $AO = BO$.

(3) Because of (1) and (2), the triangles AQO and BPO are congruent. Consequently AQ = BP.

(4) Because of (1) and (2), the triangles $AQO$ and $BPO$ are congruent. Consequently $AQ = BP$.

From (1) and (3), we are able to deduce that $AC = AQ+QC = BP+PC = BC$. Hence, the triangle is isosceles. By looking at the corners of the original triangle in a different order we can take the argument further and deduce that "Every triangle is equilateral". "Now," said Professor Forder, "who's the April Fool?"

This story from the armory of the instructor in the power-play with the instructed has been useful to me in other ways. Often we start working on a mathematical problem from an intuitive point of view. It is like starting with a sheet of paper from which much of the existing writing needs to be erased, rather than starting with a blank sheet on which the correct result is to be written. "Take care," Professor Forder seems to have been warning me over the years, "Don't assume anything you don't know".

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