Preface

This book represents an attempt to modernize and expand my previous volume 'The Numerical Analysis of Ordinary Differential Equations: Runge–Kutta and General Linear Methods'. It is more modern in that it considers several topics that had not yet emerged as important research areas when the former book was written. It is expanded in that it contains a comprehensive treatment of linear multistep methods. This achieves a better balance than the earlier volume which made a special feature of Runge–Kutta methods.

In order to accommodate the additional topics, some sacrifices have been made. The background work which introduced the earlier book is here reduced to an introductory chapter dealing only with differential and difference equations. Several topics that seem to be still necessary as background reading are now introduced in survey form where they are actually needed. Some of the theoretical ideas are now explained in a less formal manner. It is hoped that mathematical rigour has not been seriously jeopardised by the use of this more relaxed style; if so then there should be a corresponding gain in accessibility. It is believed that no theoretical detail has been glossed over to the extent that an interested reader would have any serious difficulty in filling in the gaps.

It is hoped that lowering the level of difficulty in the exposition will widen the range of readers who might be able to find this book interesting and useful. With the same idea in mind, exercises have been introduced at the end of each section.

Following the chapter on differential and difference equations, Chapter 2 is presented as a study of the Euler method. However, it aims for much more than this in that it also reviews many other methods and classes of methods as generalizations of the Euler method. This chapter can be used as a broad-ranging introduction to the entire subject of numerical methods for ordinary differential equations.

Chapter 3 contains a detailed analysis of Runge–Kutta methods. It includes studies of the order, stability and convergence of Runge–Kutta methods and also considers in detail the design of efficient explicit methods for non-stiff problems. For implicit methods for stiff problems, inexpensive implementation costs must be added to accuracy and stability as a basic requirement. Recent work on each of these questions will be surveyed and discussed.

Linear multistep methods, including the combination of two methods as predictor-corrector pairs, are considered in Chapter 4. The theory interrelating stability, consistency and convergence will be presented together with an analysis of order conditions. This will lead onto a proof of the (first) 'Dahliquist barrier'. The methods in this class which are generally considered to be the most important for the practical solution of non-stiff problems are the Adams–Bashforth and Adams–Moulton formulae. These will be discussed in detail, including their combined use as predictor-corrector pairs. The application of linear multistep methods to stiff problems is also of great practical importance and the treatment will include an analysis of the backward-difference formulae.

In Chapter 5 the wider class of General Linear Methods will be introduced and analysed. Questions analogous to those arising in the classical Runge–Kutta and Linear Multistep methods, that is questions of consistency, stability, convergence and order, are considered and explored. Several sub-families of methods, that have a potential practical usefulness, are examined in detail. This includes the so-called DIMSIM methods and a new type of method exhibiting what is known as inherent Runge–Kutta stability.