Lie algebras and some games

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Abstract

Given the important role Lie algebras play in the description of symmetries of physical systems (particularly in particle physics) and more generally throughout mathematics, the importance of their representation theory goes without saying. However, uniform constructions of representations are difficult to find and describe, even for the simplest case - finite dimensional representations of the complex simple Lie algebras.

On the other hand, the well-known classification of these Lie algebras reduces to an exercise in graph theory (the characterisation of the so-called Dynkin diagrams). So it seems natural that there might be a purely *combinatorial* approach to constructing the representations also, perhaps even a uniform construction. It is such an approach that I will speak about.

After giving a brief outline of the representation theory of these algebras, I will introduce a pair of closely related 'games' known as the *mutation* and *numbers* games. Each involves the manipulation of numerical labels on the vertices of a graph (specifically a Dynkin diagram). It will turn out that game positions for these two games can be thought of as the roots and weights in a Lie algebra and that allowed moves will correspond (in some cases) to the action of the raising and lowering operators in a representation.

Finally I will talk about an extension of these results which allows the construction of the 26-dimensional (fundamental) representation of the exceptional Lie algebra $\mathfrak{f}_4$. This description is new and, unlike previous explicit constructions, involves only integer coefficients.