

Curriculum vitae

Paul Andi Nagy

Institut für Mathematik und Informatik
Mathematisch-Naturwissenschaftliche Facultät
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Personal details

Born 4 May 1974 at Cluj-Napoca, Romania
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RESEARCH INTERESTS

Global analysis and analysis on manifolds

Geometric analysis

Geometric topology

Applications to Mathematical Physics

Geometric and non-linear PDE

EDUCATION AND QUALIFICATIONS

Hamburg University, Germany - DSc, 2008 – 2010

Habilitation thesis: *Torsion and curvature in almost-Kähler geometry*, submitted July 2010,
defence 26 January 2011

University of Savoie, France - PhD, 1996 – 2000

PhD thesis: *A separation of variables principle for the spectrum of the Laplacian on differential forms, and applications*, defended July 2001

École Normale Supérieure – University Claude Bernard Lyon 1, France - MSc, 1995 – 1996

MSc Thesis: *Metrics with harmonic spinors*, defended September 1996

École Normale Supérieure of Lyon, France – Honours (Maitrise) in Mathematics 1995 – 1996

Babes – Bolyai University of Cluj – Napoca, Romania, 1992 – 1995

Undergraduate Mathematics studies

Mathematics-Physics High-School Emil Racovita Cluj – Napoca, Romania – Baccalaureat, 1992

EMPLOYMENT

10/2010 –	Lecturer, University of Greifswald, Germany
03/2006 – 10/2010	Associate Investigator, Marsden project “ <i>Invariant Operators, Special Structures and Conformal Techniques</i> ”, Auckland, New Zealand
11/2005 – 11/2009	Research Fellow, University of Auckland and New Zealand Institute of Mathematics and Its Applications, Auckland, New Zealand
09/2003 – 11/2005	Postdoctoral Fellow, University Humboldt of Berlin, Germany within the Volkswagen Foundation project “ <i>Special geometries in Mathematical Physics</i> ”
09/2002 – 08/2003	Lecturer (Maître-Assistant), University of Neuchâtel, Switzerland , Institute of Mathematics
09/2000 – 08/2002	Lecturer (Assistant), University of Neuchâtel, Switzerland , Institute of Mathematics
09/1999 – 08/2000	Lecturer (ATER), University of Avignon, France , Department of Mathematics
1997 – 1999	Graduate Teaching Assistant, University of Savoie, France , Laboratoire de Mathématiques – LAMA
1995 – 1997	Normal School Graduate fellow, École Normale Supérieure of Lyon, France

VISITING POSITIONS AND RESEARCH FUNDING

03/2006 – 10/2010	Marsden Research grant no. 06-UOA-029 of the Royal Society of New Zealand, Associate Investigator. Project “ Invariant Operators, Special Structures and Conformal Techniques ” 450 000 NZD/approx. 250 000 EUR
09/2009 – 11/2009	Invited Research Professor, University of Turin – Politecnico of Turin, Italy
2008, 2009	Visiting Professor, University of Hamburg, Germany , several visits
11/2007– 12/2007	University of Hamburg, Germany , Kähler-Gastprofessor (Invited Professor – Kähler chair)
07/2003 – 08/2003.	Visiting research fellow, University Ludwig Maximilian, München, Germany with EDGE - “European Endeavour in Differential Geometry”
2003	With the European Project ACR OFES no. 00.0349 “Differential geometry”, University of Neuchâtel, Switzerland, 12 000 CHF/approx. 8000 EUR
1999	Within the Differential Geometry Programme, Swiss National Research Fund, University of Fribourg, Switzerland, CHF 5 000/approx. 3000 EUR
1999 – 2010	1 - 3 weeks research visits at the following institutions (travel and local costs covered by the host institution): École Polytechnique, Paris; Erwin Schrödinger Institute, Vienna; Centro di ricerca matematica Ennio De Giorgi, Pisa; Humboldt University; Politecnico di Torino; Universities of Adelaide, Auckland, Cologne, Hamburg, Nancy, Nantes.
1995 – 1997	Normal School fellowship, École Normale Supérieure of Lyon, France

PUBLICATIONS

Book chapter (invited)

1. NAGY, P.-A., [Connections with totally skew-symmetric torsion and nearly-Kähler geometry](#), in “Handbook of Pseudo-Riemannian Geometry and Supersymmetry”, ed. Vicente Cortés, European Mathematical Society Publishing House, (2010), [\(ISBN 978-3-03719-079-1\), 445-497](#)

Habilitation/DSc thesis

2. [Torsion and curvature in almost- Kähler geometry](#), 45 pgs, submitted July 2010.

PhD thesis

3. NAGY, P.A. [A principle of separations of variables for the spectrum of Laplacian acting on forms and applications](#), 2001, University of Savoie, France, 142 pages, in French

In international journals

4. Di SCALA, A. J., NAGY, P.-A., [On the uniqueness of almost- Kähler structures](#), [Comptes rendus de l'Académie des Sciences I Mathématique](#), 348 (2010), 423-425.
5. GROSJEAN J-F., NAGY P.-A., [On the cohomology algebra of some classes of geometrically formal manifolds](#), [Proceedings of the London Mathematical Society](#) (3) 98 (2009), 607-630
6. MOROIANU A. NAGY P.-A., SEMMELMANN U., [Deformations of nearly-Kähler Structures](#) [Pacific Journal of Mathematics](#), 235 (2008), no.1, 57-72
7. GOVER R., NAGY P.-A. [Four dimensional conformal C-spaces](#), [Quarterly Journal of Mathematics](#) 58 (2007), no.4, 443-462
8. BERNHARDT N., NAGY P. -A., [On algebraic torsion forms and their spin holonomy algebras](#), [Journal of Lie Theory](#) 17 (2007), no. 2, 357–377
9. BERNHARDT N., NAGY P. -A., [Spin holonomy algebras on self-dual 4-forms in \$R^8\$](#) , [Journal of Lie Theory](#) 17 (2007), no.4, 829 – 856
10. NAGY, P. -A., [On length and product of harmonic forms in Kähler geometry](#), [Mathematische Zeitschrift](#), 254 (1) (2006), 199 – 218
11. AGRICOLA, I., FRIEDRICH, T., PUHLE C., NAGY P. -A., [On the Ricci tensor in type II B string theory](#), [Classical and Quantum Gravity](#) 22 (2005), 2569 – 2577
12. MOROIANU A., NAGY P. -A., SEMMELMANN U., [Unit Killing vector fields on Nearly Kähler manifolds](#), [International Journal of Mathematics](#), 16 (3) (2005), 281-302
13. NAGY P. A, VERNICOS C., [The length of harmonic forms on a compact Riemannian manifold](#), [Transactions of the American Mathematical Society](#), 356 (2004), 2501-2513
14. NAGY, P. -A, [Rigidity of Riemannian foliations with complex leaves on Kähler manifolds](#), [Journal of Geometric Analysis](#), 13 (2003), no.4, 659-667.
15. NAGY P. -A., [On Nearly Kähler geometry](#), [Annals of Global Analysis and Geometry](#), 22, (2002) 167-178
16. NAGY P. -A., [Nearly Kähler geometry and Riemannian foliations](#), [Asian Journal of Mathematics](#), 6 (3) (2002), 481-504
17. (2008) NAGY P.-A., [Skew-symmetric prolongations of Lie algebras and applications](#), arXiv:0712.1398, 28 p.
18. (2010) CHIOSSI S., NAGY P.-A. [Complex homothetic foliations on Kähler manifolds](#) arXiv:1008.4954, 12p , submitted

National seminar proceedings

19. NAGY P. A., *Le spectre du laplacien agissant sur les p -formes différentielles d'un fibré en cercles*, Séminaire de théorie spectrale et géométrie, Grenoble, volume 17, (1999), 185-195

Didactical papers in a national peer-reviewed journal

20. NAGY P.-A., *The identity $x^{n+1}=x$ in rings*, Didactica Mathematica, 10 (1994), 91-94

21. NAGY P.-A., *On the convergence order of the real sequence $1/(2\ln 2)+\dots+1/(n \ln n) - \ln[\ln n]$* , Didactica Mathematica, 10 (1994), 95-98

22. NAGY P.-A. *On a class of real sequences*, Didactica Mathematica, 10 (1994), 77-90

In preparation

* with LEISTNER Th. (Adelaide) Classes of diagonalisable metrics in dimension 4

* with GOVER R. (Auckland) Cotton-Weyl structures on Kähler manifolds 21.

* with SVENSSON M (Odense) Harmonic morphisms on Kähler manifolds

* several papers with S. Chiossi (Torino), U. Semelmann (Stuttgart) and A. Moroianu (Paris)

TEACHING

Undergraduate teaching experience

2010 S1	Analysis I (tutorials), University of Greifswald
2009 S2	Mathematics for Physicists III (tutorials), Hamburg University
2008/2009 S1:	Differential Geometry II (Lie Groups and symmetric spaces) (course), Hamburg University, Germany
2007/2008 S1:	Mathematics for Physicists II (tutorial) Hamburg University
2006 S1:	Geometry and Topology (course and tutorial), University of Auckland, New Zealand
2004/2005 S2:	Introduction to General Relativity (tutorials), Humboldt University of Berlin, Germany
2000 – 2003	Differential Geometry and Differential and Integral Calculus I, II (tutorials), University of Neuchâtel, Switzerland
1999 – 2000	Methodologies in Mathematics (large class course and exercises), Mathematics for Earth and Life Sciences (large class exercises), University of Avignon, France
1996 – 2000	Analysis I, II, Algebra I, II, Differential Geometry and Mathematics for Economics , etc., (tutorials) University of Savoie (Chambéry and Annecy campuses)
1997 – 1999	Preparatory School (Classes préparatoires MPS, MPSI): small group tutorials and individual examinations, Lycee Vaugelas, Chambéry and Lycee Aux Lazaristes, Lyon, France

Graduate and postgraduate teaching

2010 S2	Analysis on Manifolds and Differential Geometry (part of the course), University of Auckland, New Zealand
2009 S2	Lie Groups and Lie Algebras (course), University of Auckland, New Zealand Symplectic Geometry (course and tutorials), Hamburg University
2008 S2:	Analysis on Manifolds and Differential Geometry (course), University of Auckland, New Zealand
2007/2008 S1:	Introduction to Symplectic Geometry (course), Hamburg University, Germany

- 2007 S1: **Complex Analysis** (course), University of Auckland, New Zealand
- 2007/2008 S1: **Introduction to $U(n)$ and $SU(n)$ structures** (Almost Hermitian Geometry) (postgraduate invited course based on own research), Hamburg University
- 2006 S2: **Analysis on manifolds and Differential Geometry** (course), University of Auckland, New Zealand
- 2004/2004 S1: **Introduction to Kähler Geometry** (course), Humboldt University of Berlin, Germany
- 2002/2003 S2: **Nearly- and almost-Kähler Geometry** (postgraduate course), University of Neuchâtel, Switzerland
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SUPERVISION

PhD theses

- 2005 – 2008 Niels Bernhardt "**Parallel spinors, torsion forms and some classes of geometric structures**", University of Auckland. (main supervisor; the thesis was funded by a RSNZ Marsden grant. A. Rod Gover was co-supervisor)
- since 2008 Yuri Vyatkin "**Bernstein-Gelfand-Gelfand complexes**" University of Auckland. (co-supervisor; thesis funded by a University of Auckland scholarship, A Rod Gover is the main supervisor)

Master thesis

- 2004 – 2005 Christof Thies, "**Half-Flat $SU(3)$ structures**", Diploma of the Humboldt University of Berlin.

Honours dissertations and (under)graduate research projects

- 2006 Geon Oh, "**PSU(3)-structures on 8-manifolds**", reading project, University of Auckland
- 2007 Geon Oh, "**Differentiable structures**", BSc(Honours) dissertation, University of Auckland
- 2007 Matthias Drews, "**Self-duality in dimension 4**", reading project, University of Auckland
- 2010 Jonas Grabbe, "**Killing spinors and Gray manifolds**", Bachelor dissertation, Hamburg University
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TEACHING STATEMENT

I am a well-rounded mathematician, with a rich research and teaching portfolio.

I am able and willing to teach and tutor numerous topics in Analysis, Algebra and Geometry, as well as other Pure and Applied Mathematics topics.

I enjoy new teaching experiences and supervisory work. I have a keen interest to develop school outreach activities, and a good record of attracting new students at all levels.

My teaching experience covers a broad spectrum of student backgrounds and teaching levels, as well as four languages of instruction (English, French, German, Romanian).

I have taught at all levels: from preparatory school and service undergraduate courses, to invited postgraduate courses based on my research, which were attended by PhD students, research fellows and Faculty.

I delivered courses and exercises in various teaching arrangements: from sole lecturer and course coordinator, responsible for developing course material, assignments, examinations etc, to member of a lecturing team (delivering - for instance - $\frac{1}{2}$ of the course) or tutor for another lecturer.

I enjoyed the opportunities of developing a course from scratch (and sometimes learning part of the covered contents along the path) or revising existing course material.

I successfully supervised several students and coordinated reading seminars for students. I gave numerous seminar and conference talks, on invitation or at the home institution

Background

Masters degree in Mathematics from the École Normale Supérieure of Lyon (ENS – Lyon, France, 1996) and PhD from the University of Savoie (France, 2001).

As a

- Mathematics Olympiad contestant (Romania, 3rd prize in 1990, 1991),
- undergraduate student with Babes-Bolyai University of Cluj-Napoca, Romania (1992-1995), and
- graduate student with the École Normale Supérieure of Lyon, France (1995-1997),

I have benefited from a tradition fostering a very sound mathematical background.

Thanks to this background, I have developed a taste for accompanying (under)graduate students through their individual study of Mathematics.

However, my interest for proposing interesting mathematical problems was a constant of my mathematical development: during high-school years, some of my problems were published in the famous Romanian *Gazeta Matematica*. As an undergraduate student, three of my problems were published as mathematics education articles in a national journal. As a lecturer, I enjoyed problem-solving tutorials and the supervisory experience.

Teaching objectives

- to create a stimulating environment for students to build up a sound mathematical background
- to adapt teaching strategies to students background and interests
- to encourage mathematical vocations
- to maintain up-to-date course material and exercises data base

Personal development objectives in teaching

- to broaden the existing portfolio by undertaking teaching in new subjects
- to develop skills related to the use of new technologies in the class

INVITED TALKS

Conferences and workshops

- Invited talk, Extremal metrics: evolution equations and stability/ Métriques extrêmes : équations d'évolution et stabilité, CIRM, Luminy, France, 7-12 February 2011
- [Conference on Kähler and related geometries](#), Nantes, France, 9-13 November 2009
Talk: "*The torsion and curvature of some classes of almost Kähler manifolds*"
- [Turin Differential Geometry Day](#), Turin, Italy, 17 July 2009
Talk: "*Hermitian Structures on Kähler surfaces*"
- [Geometric Spectral Theory](#), Neuchâtel, Switzerland, 8-12 June 2009
Talk: "*Numerical form eigenvalues from Einstein deformations*"
- [Holonomy Groups and applications in String theory](#), Hamburg, 14-18 July 2008
Talk "*Skew-symmetric prolongations of Lie algebras and applications*",
- [Parabolic geometry and PDE](#), Auckland, 4-8 August 2008
Talk: "*Skew-symmetric prolongations of Lie algebras*",
- [Pseudo-Riemannian Geometry and Super symmetry](#), Strasbourg, France, 22-24 September 2005
Talk: "*Symmetries and deformations of nearly-Kähler manifolds*"
- [9th International Conference on Differential Geometry and its Applications](#), Prague, August 30 - September 3, 2004
Talk "*Algebraic reduction and local structures of certain almost-Kähler manifolds*",
- [Geometric and analytic problems related to Cartan connections](#) Program, Erwin Schrödinger Institute, Vienna, Austria, April 2004.
Talk: "*Nearly Kähler manifolds with symmetries*"
- [Differential Geometry and Topology Programme](#), Centro di ricerca matematica Ennio De Giorgi, Pisa, Italy, November 2004.
Talk: "*Reduction of Nearly-Kähler 6-manifolds*"
- [Black-Sea European Differential Geometry Endeavour meeting](#), Varna, Bulgaria, July 2003
Talk: "*Torsion in almost Kähler geometry*".
- [Geometry of low-dimension manifolds and special geometries](#), International Centre of Research in Mathematics (CIRM), Luminy, France, 3-7 June 2002
Talk: "*Nearly Kähler geometry and related structures*",
- Meetings of the 3rd Romand cycle of Mathematics, Neuchâtel, Switzerland, 2001
Talk: "*Nearly Kähler geometry and Riemannian foliations*"
- Autumn Meeting of the Swiss Mathematical Society, Yverdon, Switzerland, 19-10-2001
Talk "*The differential form spectrum of a circle bundle*",
- Harmonic analysis and manifolds analysis, CIRM, Luminy, France, 31 May - 4 June 2009
Talk: "The differential form spectrum of a circle bundle"

Research seminars

- **Universities of Cologne, Regensburg, Stuttgart** – titles to be confirmed, 11-2010 and 12-2010
- **University of Adelaide, Australia**: "On the uniqueness of almost- Kähler structures" 28-05-2010
- **Politecnico di Torino, Italy**: "Torsion and curvature in almost-Kähler geometry", 08-10-2009
- **Hamburg University, Germany**: "Manifolds with many symplectic structures", 20-05-2009
- **Hanover University, Germany**, "Kähler structures on Hermitian surfaces", 7-05-2009
- **École Polytechnique, Paris, Arthur Besse Seminar**: "On systems of symplectic forms", 13-01-2009
- **University of Cologne, Germany**: "On systems of symplectic forms", 5-12-2008
- **University of Cologne, Germany**: "Prolongations of Lie Algebras and Applications", 26-11-2007

- **Trinity College, Dublin, Ireland:** “A class of equations in type II B string theory”, 05-10-2005
- **University of Nancy - Institute Elie Cartan, Nancy, France:** “Nearly Kähler manifolds with symmetries”, 22-05-2005
- **University of Auckland, New Zealand:** “Nearly Kähler manifolds”, 16-03-2005
- **Warsaw University - Department of Mathematical Methods in Physics, Poland:** Foliations of compact Kähler manifolds, 13-11-2003
- **Humboldt University Berlin - Institute of Mathematics, Germany,** “Nearly Kähler geometry”, 15-05-2002
- **Ludwig Maximilians University Munich - Institute of Mathematics, Germany** “Nearly Kähler geometry and applications”, 21-06-2002
- **University of Nancy - Institute Elie Cartan, Nancy, France:** “Torsion in almost Hermitian geometry”, 12-12-2002
- **Spring Meeting of Differential Geometry, Nîmes, France:** “The differential form spectrum of a circle bundle”, France, 06-09-2000
- **Institut Fourier, Grenoble, France - Seminaire de géometrie et théorie spectrale:** “Le spectre du laplacien sur les formes différentielles d’une variété pseudohermitienne”, 1999

Seminar talks at the home university

Based on own research unless specified

University of Auckland - Geometry and Analysis seminar series

- Toric nearly Kähler manifolds, 20-03-2009
- Symmetry degree from framing, *survey 07-03-08*
- Classification of Berger algebras, *survey 03-08-2007*
- Prolongations of Lie algebras and applications, *23-07-2007*
- Formal curvature tensors and Berger algebras, *survey, 29-06-2007*
- Geometrically formal manifolds, *9-03-2007*
- Principles of separation of variables in the spectrum of the Laplacian, *25-08-2006*

Humboldt University of Berlin – Global analysis and differential geometry seminar series

- Foliations on Kähler manifolds, *19-10-2003*
- Around Ströminger's type II string equations, 04-2004
- Local structures of certain almost-Kähler manifolds,, 26-10-2004
- On symmetries of NK-3 folds, 20-04-2004

University of Neuchâtel

- Geometric formality, 03-05-2003
- Length of harmonic forms, literature survey 04-2003
- Seiberg-Witten invariants, hyperbolic geometry and harmonic forms, literature survey 03-2003

University of Avignon

Le spectre du laplacien sur les formes différentielles d’une variété pseudohermitienne, 1999

University of Savoie

The differential form spectrum of a circle bundle, *series of talks, 1997-1998*

OTHER PROFESSIONAL ACTIVITIES

Referee for several A and A* international journals

Conference and seminars organisation

- Differential geometry graduate seminar, Hamburg University, Germany, spring 2009
 - Geometric Analysis working group seminars, University of Auckland, winter 2008
 - Co-organisation of the “2008 NZMRI Conference on Conformal Geometry”, Nelson, New Zealand, 6-12 Jan 2008
 - Co-organisation of the Workshop "Special Geometries in Mathematical Physics". Kühlungsborn, Germany, March 2006.
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AWARDS

- 1991 National contest of problem solvers of the “Gazeta Matematica” journal, Romania, honourable mention
 - 1991 Regional Olympiad of Mathematics, Cluj, Romania 1st prize
 - 1992 National Olympiad of mathematics, Romania, 3rd prize;
 - 1993 National university contest "Traian Lalescu", Romania, 2nd prize
 - 1994 National university contest "Traian Lalescu" Romania, 1st prize
 - 1994 International academic contest "Traian Lalescu", Romania-URSS, 3rd prize
 - 1992-1995 Merit grant of the Babes-Bolyai University for excellence of studies
 - 1995 1st rank at the admission contests as Fellow of the French Normal Schools (ENS), for a 2 year graduate fellowship.
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REFEREES

- Prof. [A. Rod Gover](#), Department of Mathematics, University of Auckland
gover@math.auckland.ac.nz
- Prof [Paul Gauduchon](#), Ecole Polytechnique, France paul.gauduchon@math.auckland.ac.nz
- Prof [Vicente Cortés](#), Hamburg University, cortes@math.uni-hamburg.de
- Prof [Andrew Swann](#), University of Southern Denmark, Odense, swann@imada.sdu.dk
- Prof. [Uwe Semmelmann](#), Mathematics Institute, University of Cologne,
uwe.semmelmann@math.uni-koeln.de

STATEMENT OF RESEARCH AND RESEARCH PROJECTS

PAUL-ANDI NAGY

The theory of geometric structures on Riemannian manifolds is unifying the themes described in this text. Using techniques and ideas from global and geometric analysis and holonomy theory for connection with torsion I was able to solve problems broadly grouped in the following categories:

- (i) classification of geometries with torsion on manifolds and geometric foliations;
- (ii) behaviour of length and product of harmonic forms of a Riemannian metric;
- (iii) generalisation of Einstein manifolds.

Applications include solutions of equations in mathematical physics, in particular those involved in the type IIB string equations and supergravity.

RESEARCH INTERESTS BY PROJECT

Almost-Hermitian geometry. My early research interest has been in nearly-Kähler geometry which is one of the 16 classes of almost Hermitian manifolds in the Gray-Hervella classification. This class of structures unifies several important themes and constructions in differential geometry ranging from Kähler to quaternion-Kähler geometry via twistor theory, as developed by Penrose, Bérard-Bergery and Salamon. A significant part of the research in this domain is driven by considerations originating in the branch of theoretical physics known as string theory; in fact nearly Kähler manifolds of dimension 6 are at the center of fundamental conjectures in mirror symmetry, as outlined in the Yau's programme for non-integrable geometries.

The mathematics of the geometries with torsion, including nearly-Kähler structures, is a new direction, that has expanded considerably during the last two decades. Contrary to the integrable (Riemannian) case, no general tools - such as the deRham or Berger theorems - are available. These have to be developed for each of the cases under scrutiny.

Date: August 10, 2010.

It has been my contribution to the subject to achieve this for nearly-Kähler manifolds. In [2] I have proved a structure result showing that the main building blocks are nearly-Kähler structures supported by Eells-Salamon twistor spaces over positive quaternion-Kähler manifolds, 3-symmetric spaces and nearly-Kähler structures in dimension 6. The class of 3-symmetric spaces consists in homogeneous spaces with a geodesic symmetric of order 3, and therefore generalises the notion of symmetric space where an involution is required.

The main ingredient has been to show that a holonomy reduction for the canonical Hermitian connection (which has torsion and therefore does not obey the deRham splitting theorem) can be brought algebraically to normal form. Normal forms correspond geometrically to complex (w.r.t to a Kähler structure) totally geodesic, Riemannian submersions and in this case the holonomy of the canonical connection is related to the Riemannian holonomy of the base space. The argument is completed by using the Berger holonomy classification theorem and some global stability arguments on the leafs of foliation above.

Applications.

- The proof of the long-standing Gray-Wolf conjecture (1969) asserting that a nearly Kähler 3-symmetric space has to be naturally reductive as a homogeneous space.

In [2] I have proved the Gray-Wolf conjecture in all dimensions but 6. This last case has been recently solved by J.-B. Butruille. The main result in [2] also yields the full classification of totally geodesic, Riemannian foliations on Kähler manifolds.

- In the compact case, by examining the relevant transverse cohomology complex I have also derived in [3] structure results for Riemannian (non-totally geodesic) foliations with complex leaves on Kähler manifolds.
- Nearly-Kähler geometry makes sense when allowing the metric to be pseudo-Riemannian, that is to have indefinite signature. In this case the classification programme has been taken up by research groups in Hamburg, Hannover, Tenerife, Cordoba.
- Nearly-Kähler structures in dimension 6 are typical examples of Calabi-Yau manifold with torsion; they admit an $SU(3)$ -structure (and therefore have vanishing first Chern class) and support an Einstein metric.

Together with A. Moroianu and U. Semmelmann we have obtained in [6] a characterisation of nearly-Kähler 6-manifolds that admit a unit Killing vector field.

- A large class of examples with singularities admitting a homothetic Killing field has been recently obtained in work by Fernández et al.
- In joint work with Moroianu and Semmelmann [11] we have computed the infinitesimal deformation space of a nearly-Kähler metric in dimension 6. Based on this work the last named authors have also proved the existence of infinitesimal deformations on the flag manifold $F(1, 2)$.
- One of the basic features of nearly-Kähler geometry is to have its intrinsic torsion tensor parallel w.r.t the canonical Hermitian connection. A suitable extension, to the larger class of almost Hermitian structures with totally skew-symmetric covariant Nijenhuis tensor, has been described in the first part my book chapter [13] on nearly-Kähler geometry by means of the notion of Hermitian Killing forms. This is a new class of first order linear PDEs which has been shown in [13] to govern the geometry of the structures above.

Length and product of harmonic forms. Given (M^n, g) a closed Riemannian manifold, one defines the space of harmonic p -forms by $\mathcal{H}^p(M, g) = \text{Ker}\Delta \cap \Lambda^p M$ where Δ is the Laplace operator w.r.t the metric g . It is used to describe the topology of M by means of the linear isomorphism

$$(0.1) \quad H_{DR}^p M \cong \mathcal{H}^p(M, g)$$

provided by Hodge theory; note this is strongly depending on the metric g .

The length of harmonic forms contains additional metric information as shown by the generalised isosystolic inequalities of Bangert et Katz:

$$(0.2) \quad stsys_1(g) \cdot sys_{n-1}(g) \leq \gamma'_{b_1(M)} \cdot vol(g).$$

Here the stable 1-systole $stsys_1(g)$ is defined using the stable norm, $sys_{n-1}(g)$ is the inf of $(n-1)$ -non-separating hypersurfaces in M and $\gamma_{b_1(M)}$ is the Bergé-Martinet constant of M . The inequality (0.2), which improves previous results of Lévy and Gromov, connects information on homology (coming from the stable norm) to the geometry of the manifold.

Inequalities (0.2) are saturated by Riemannian metrics having all harmonic 1-forms of pointwise, constant length. In collaboration with C. Vernicos [4] we have classified such metrics when $b_1(M) = n-1$.

In a further paper [7] I have shown that (0.2) are always strict for hyperbolic or Kähler metrics; at the time, this was an open problem.

Riemannian metrics with harmonic forms of constant length also turn up in the following context, introduced in 2001 by D. Kotschick. The metric g is formal if the exterior product of harmonic forms remains harmonic, that is $\mathcal{H}^*(M, g)$ is a sub-algebra of $(\mathcal{A}M, \wedge)$.

Understanding the properties of formal metrics is an important open problem in geometric analysis: it is the geometrisation of the notion of topological formality in the sense of Sullivan which implies that the rational homotopy type of the manifold is a formal consequence of its co-homology ring. Fundamental work of Deligne and Griffiths shows that Kähler manifolds are topologically formal.

In [7] I have studied formal Kähler metrics and showed that the corresponding $\mathcal{H}^2(M, g)$ is spanned by compatible symplectic forms. Under various curvature assumptions rigidity results are given and the geography of Chern numbers explored in the same work.

Building up on these results, we prove in collaboration with J.-F. Grosjean [12] that formal Sasakian metrics can exist only on real co-homology spheres and also that formal Kähler metrics have Todd genus

equal to 1. In small even dimensions we were able to describe the second co-homology as follows:

Theorem 0.1. [12] *Let M be closed of dimension 6, with real co-homology algebra non-isomorphic to that of $T^3 \times S^3$. If $b_1(M) \neq 1$ and $b_2(M) \geq 2$ any formal metric on M admits a compatible symplectic form.*

This is proved by topological means when $b_2(M) \geq 3$. When $b_2(M) = 2$ we first show that $b_3(M) = 6$ if M admits a formal metric without compatible symplectic forms. The conclusion comes from the local analysis of the differential system induced by the harmonic 3-forms, along the two naturally associated foliations. Note that the case of dimension 4 has been already treated by Kotschick, using Seiberg-Witten theory.

Theorem 0.1 is expected to shed light on the relation between (topological) formality and symplectic geometry.

Holonomy of spinorial and torsion connections. Let (M^n, g) be Riemannian. The classification of linear connections on M by means of the symmetries of their torsion tensor goes back to É. Cartan. We are interested in the case when D is metric, that is $Dg = 0$ and has totally skew-symmetric torsion. Connections in this class, largely present in string theory, are of the form

$$(0.3) \quad D = \nabla + T$$

where ∇ is the Levi-Civita of g and T belongs to $\Lambda^3 M$. There are no general results describing the holonomy representation of such connections, as opposed to the torsion free or Riemannian case. The natural problem to look at is therefore

Problem 1. *Let (M^n, g) be Riemannian and let $G \subset O(n)$ be a Lie sub-group. What are the necessary and sufficient conditions on g and G that ensure the existence of a connection of type (0.3) such that*

$$(0.4) \quad \text{Hol}(D) \subseteq G$$

at some point of M ?

A particular case of this problem is the Strominger model in type IIB string theory. It requires the presence of a spinor field ψ , a dilaton function δ and a connection D of type (0.3) such that

- (1) $D\psi = 0$;
- (2) $(T - 2d\delta)\psi = 0$ par rapport à la multiplication de Clifford;
- (3) $d^*T = 2\text{grad}\delta \lrcorner T$;
- (4) $\text{Ric}^D + \frac{1}{2}d^*T + 2\nabla(d\delta) = 0$

where Ric^D is the Ricci tensor of D . We have discovered in [5] that having constant dilaton forces the solution to be unique and of Riemannian type, that is $T = 0$.

Recently I have worked out the uniqueness aspect of problem 1.

Theorem 0.2. [15] *If G acts irreducibly on \mathbb{R}^n a connection of type (0.3) with $\text{Hol}(D) \subseteq G$ must be unique, unless (G, \mathbb{R}^n) is the adjoint representation of G .*

The tools needed for the proof include the classification of Berger algebras due to S. Merkulov and L. Schwachhöfer. In [15] I have obtained a full solution to problem 1 in the case of vectorial torsion, that is contained in $\Lambda^1 M \subseteq \Lambda^1 M \otimes \Lambda^2 M$.

In collaboration with N. Bernhardt, who has studied for PhD (and completed in 2009) under my supervision, we looked at the linear generalisation of problem 1 to spinors in order to obtain explicit examples.

For any inhomogeneous, linear form, T in $Cl(\mathbb{R}^n)$ consider

$$\mathfrak{g}_T^* = Lie\{X \lrcorner T : X \in \mathbb{R}^n\}.$$

The connection $\nabla + \frac{1}{2}T$ acting on spinors of $(\mathbb{R}^n, flat)$ has holonomy algebra $[\mathfrak{g}_T^*, \mathfrak{g}_T^*]$. Such connections and especially those admitting parallel spinors are systematically considered in supergravity theories. The Lie algebras \mathfrak{g}_T^* are extremely complicated objects, fact due to the polynomial nature of their generators. In [8] we have discovered that mild conditions on T , such as self-duality in the Clifford sense force these algebras to be semi-simple. Explicit computations, for spinor squares, have been produced in the same article. A full classification of \mathfrak{g}_T^* when T in $\Lambda^4(\mathbb{R}^8)$ is self-dual has been obtained in [9].

Generalisations of the Einstein condition. Let (M^n, c) be a conformal manifold with a Weyl structure D , that is a torsion free connection that preserves the conformal class. P. Gauduchon proved in 1995 that the Cotton-York tensor of D vanishes if and only if for any metric $g \in c$ there exists a vector field ζ such that

$$(0.5) \quad C + W(\zeta, \cdot) = 0.$$

Here C and W are, respectively, the Cotton-York tensor and the Weyl tensor of g . Therefore (0.5) naturally generalises to the conformal setting the class of Riemannian manifolds with harmonic Weyl tensor. A typical example is provided by locally conformally Einstein metrics. Together with R. Gover we show that

Theorem 0.3. [10] *Let (M^4, g) be Riemannian such that (0.5) is satisfied for some vector field ζ on M . If M is compact the one form ζ^\flat is closed, in other words g is locally conformal to a metric with harmonic Weyl tensor.*

When compactness is not assumed the result holds on the open set where the Weyl tensor does not vanish. This is optimal by an old result of Tanno; every Sasaki-Einstein manifold (and hence of odd dimension) satisfies (0.5) when ζ is taken to be the Reeb vector field of the structure. The proof of the local result is achieved by constructing orthogonal complex structures on open sets of M ; to conclude in the global case we establish and then use that g has the analytic continuation property.

In work in progress [1] the structure of the C-space equation on Kähler manifolds is currently explored.

CURRENT PROJECT: THE METRIC SYMPLECTIC PROPERTY

A symplectic structure on manifold M^{2m} consists in a closed differential two form ω which is everywhere non-degenerate. Any symplectic form can be calibrated by almost complex structures J , that is endomorphisms of TM with $J^2 = -1$, $\omega(J\cdot, J\cdot) = \omega$ and such that $g_J = \omega(\cdot, J\cdot)$ is a Riemannian metric. Deep conjectures by Donaldson assert the existence of a distinguished (extremal for the generalised Calabi functional) calibration for a given symplectic form should have analogous algebro-geometric interpretations as it is the case in Kähler geometry.

The current project aims to classify Riemannian metrics that are compatible with a symplectic form and are subject to Kähler type curvature conditions, without being integrable; in particular they are extremal for both the total scalar curvature and energy functionals. In dimension 4 those have been studied in a series of works by Apostolov et al.

In my habilitation thesis [1] new techniques are being developed to treat the case of arbitrary dimensions. New non-homogeneous examples, including Einstein metrics are given and the Goldberg conjecture is proved under these assumptions. The latter asserts that on a compact manifold every Einstein metric compatible with a symplectic form is Kähler; it is still wide open in the case of negative scalar curvature.

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