

DEPARTMENT OF MATHEMATICS

MATHS 190

Tutorial 3

Tutorials in Maths 190 are **collaborative tutorials**. You should work in groups of 3 or 4 students, discussing the situations and puzzles listed below, or issues arising from lectures. Part of your final mark depends on your participation in tutorials.

1. Find:

(a) $54 \pmod{7}$

(b) $8 \times 6 \pmod{7}$

(c) $8 \times 9 \times 10 \pmod{7}$

(d) $8^2 \pmod{7}$

(e) $8^{423} \pmod{7}$

(f) $9^{301} \pmod{7}$

If you used a calculator for any of these, go back and think about how to do it without a calculator.

2. (a) Tip Top Jelly Tip Ice Cream (2L) has the bar code number 9 414932 189725. Check that this number is correct.

(b) “The Science of Discworld” by Terry Pratchett, Ian Stewart and Jack Cohen has the ISBN number 0 091 86515 8. Check that this number is correct.

3. A bank allocates account numbers with a check digit system. Its account numbers are eight digits, d_1 to d_8 . They choose the last digit so that

$$d_1 + 3d_2 + 7d_3 + 3d_4 + d_5 + 3d_6 + 7d_7 + 3d_8 \equiv 0 \pmod{10}.$$

Find the last digit of the bank account number 3050923_.

4. Prove that $\sqrt[3]{2}$ is irrational.

5.

Write up your answer to this question and hand it in with your answers to Assignment 2 (due August 26th). See below for instructions on writing tutorial reports. Don't forget to write down the names of the people in your tutorial group, so that you can acknowledge your collaborators in your report.

Show that the product of two rational numbers is rational. What about the sum of two rational numbers? What about the product of two irrational numbers? Make sure you justify your answer in detail.

6. **Harder.** Consider the number with the decimal expansion $0.12345678910111213\dots$, that is, the number formed by writing the natural numbers in order. Is this rational or irrational? Use a proof by contradiction to carefully prove your result.

What about the number $0.246810121416\dots$ (the number formed by writing all the even natural numbers in order)? Is this rational or irrational?

Writing up tutorial questions

A written solution for question 5 should be handed in for marking with Assignment 2 (due August 26th). Instructions on how to hand in your assignment are on the Assignment 2 question sheet.

In your solution, you should include:

- the names of the people you discussed this with in your tutorial group;
- a clear statement of your solution to the puzzle;
- a clear explanation (in one or two paragraphs) or how you arrived at this solution; and,
- a statement of any assumptions you had to make in obtaining your answer.