Maths 190 Lecture 7

Topic for today:
Infinity

Question of the day:
What comes to mind when you hear the word "infinity"? Write down your thoughts and questions.

How can we find out if two groups (e.g., of people, tennis balls, sweets, etc) have the same number of members?

What do we do if we can't count the members of the group?

Some notation:

When we write $\{1, 2, 3, \dots\}$ we mean the set consisting of the numbers 1, 2, 3 and all the other numbers following in the sequence.

The curly brackets indicate that we are talking about a set. The dots indicate that we continue the sequence. Dots with no number after them means the sequence goes on forever.

Examples:

$\{1, 3, 5, \dots 13\}$ means the set of odd numbers from 1 up to 13.

$\{1, 3, 5, \dots\}$ means the set of all positive odd numbers.

$\{1, 3, 5, 7, 9 \dots\}$ also means the set of all positive odd numbers.

Examples of one-to-one correspondence:

Find a one-to-one correspondence between the elements in the following pairs of sets.

1. $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

   $\{2, 3, 4, 5, 6, 7, 8, 9, 10\}$

2. $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

   $\{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

3. $\{1, 2, 3\}$

   $\{5, 6, 7, 8\}$

4 $\{1, 2, 3, 4, 5, 6, 7\}$

$\{-3, -2, -1, 0, 1, 2, 3\}$

5 $\{1,\ 2,\ 3,\ 4,\ 5, \square \ldots \square\}$

$\{\square, \ldots \square, -2, -1, 0, 1, 2, \square, \ldots, \square\}$

In this example we can assume that there are the same number of elements in both sets but we do not know how many. The boxes are covering up some numbers.

Important Fact:

The only way two sets can be the same size is if their elements can be put in one-to-one correspondence.

Some definitions

A non-empty set D is said to be **finite** if there is a natural number n such that there is a one-to-one pairing between the elements in the set D and the elements of the set $\{1, 2, 3, \ldots, n\}$.

An **infinite set** is a set that is not finite.

Another definition

Two sets have the same **cardinality** if there is a one-to-one correspondence between the elements of one set and the elements of the other.

Try this exercise in pairs:

Compare the cardinality of the following sets. Which set of each pair is larger? If the sets are the same size as each other, find a one-to-one correspondence.

- $\{1, 3, 5, 7, \dots\}$

  $\{2, 4, 6, 8, \dots\}$

- $\{1, 2, 3, \dots\}$

  $\{2, 3, 4, \dots\}$

- $\{1, 2, 3, \dots\}$

  $\{1, 4, 9, 16, 25, \dots\}$

When comparing two infinite sets, some pairings between the two sets might be one-to-one correspondences while other pairings are not.

For example, compare the sets

$\{2, 3, 4, \dots\}$

$\{1, 2, 3, \dots\}$

Failure to find a one-to-one correspondence between infinite sets does not necessarily mean that there is no one-to-one correspondence.

Important ideas from today:

- The only way two sets can be the same size is if their elements can be put into a one-to-one correspondence.
- A non-empty set is finite if there is a natural number n such that there is a one-to-one pairing between the elements in the set and the elements in $\{1, 2, 3, \ldots, n\}$
- An infinite set is a set that is not finite.
- For infinite sets, some pairings may give one-to-one correspondences even though other pairings do not. Having a pairing that is not a one-to-one correspondence does not mean that there are no one-to-one correspondences.

For next time

- Read §3.1 and §3.2 in the textbook.
- Try some Mindscapes at the end of §3.1 of textbook.
- Ask a friend what ideas come to mind when they think of infinity. Do they have similar ideas to you? Do either of you think there could be different sizes of infinity?