Department of Mathematics

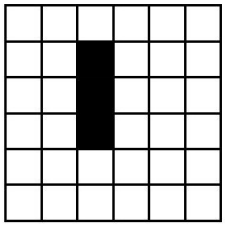
Maths 190 Assignment 4

May 19, 2010 Due: 4pm, Monday May 31, 2010

- Put your completed assignment in the appropriate box in the basement of the Mathematics/Physics Building **before** 4pm on the date due.
- Late assignments or assignments placed in the wrong box will not be marked.
- Your assignment **must** be accompanied by a blue Mathematics Department coversheet. Copies of the coversheet are available in the basement.

1. (6 marks) Consider the game "immortal life" which has the same birth rules as the game of life, but no death rules.

   Given the following initial population, what is the population in the next 3 time steps? Draw your answer on the solution sheet.



2. (6 marks) Let $c > 0$ and $0 < P_1 < 1$ be real numbers. Define a sequence of values generated by the formula

$$P_{n+1} = P_n + cP_n(1 - P_n).$$

   (a) Using Excel or any other computer programme, explore what happens with this sequence for the three values $c = 2$, $c = 2.5$ and $c = 3$ and various starting points (more precisely, make small changes to P_1 and see if they lead to large or small changes in the long term behaviour of the sequence).

   Very briefly describe what you have seen.

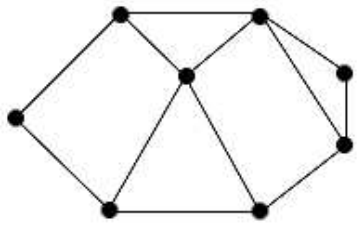
   (b) For which values c does the sequence exhibit chaotic behaviour?

3. (5 marks)

   (a) Explain why the below graph does not have an Euler circuit.

   (b) What is the minimum number of edges to be added to the graph before it has an Euler circuit?

   (c) On the picture on the attached page, draw these additional edges.



4. (5 marks) A soccer ball is made up of pentagons and hexagons. Let n be the number of pentagons in a soccer ball and m the number of hexagons.

   (a) Explain why $m = 5n/3$.

   (b) Consider the graph, drawn on the surface of a sphere, whose edges are the joins between pentagons and hexagons and whose vertices are corners of pentagons. Let V, E and F be the number of vertices,edges and faces of this graph.
   Explain why $V = 5n$, $E = 15n/2$ and $F = 8n/3$.

   (c) Use the Euler characteristic to solve for n and hence determine the number of pentagons and hexagons in a soccer ball.

5. (3 marks) Draw a graph on the surface of a torus such that the Euler formula $V-E+F = 2$ does not hold.

Tutorial write up: Remember to hand in with your assignment your written solutions to Question 4 of Tutorial 9 (5 marks), Question 4 of Tutorial 10 (5 marks) and Question 5 of Tutorial 11 (5 marks).

Assignment 4: Solution Sheet