DEPARTMENT OF MATHEMATICS

MATHS 260 **Tutorial 3**

The aim of this tutorial is to (i) practice using the Existence and Uniqueness Theorems to find out when solutions to DEs exist, and (ii) to practice modelling skills.

1. Consider the initial value problem

$$\frac{dy}{dt} = y^3, \quad y(0) = 0.5.$$

   (a) Use the Existence and Uniqueness theorems to show that this IVP has a unique solution. Check your answer with your tutor to be sure you have done this correctly.

   (b) Use *dfield* to plot an approximate solution to the IVP. For what values of t do you think the solution exists?

   (c) Solve the IVP analytically (i.e., find a function $y(t)$ that satisfies the IVP) and check your answer to (b).

2. This question is about the effect of harvesting on a population of bacteria.

   (a) Write down a differential equation to model the growth of a population of bacteria, assuming that a small population will initially grow at 10% per day, but the carrying capacity of the environment (i.e., the maximum size of population that can be be sustained) is 50,000. Scale your dependent variable so that a value of 1 represents a population of 1000 individuals.

   (b) Find the equilibrium solutions for your model. Draw a phase line.

   (c) Modify your model to include the effect of 1000 bacteria being removed (i.e., harvested) every day.

   (d) Use *dfield* to draw the slope field for your modified model, and estimate the new equilibrium populations. Draw a phase line. Be sure to make the ranges on your plot window large enough - otherwise you will get the wrong answer.

   (e) Now modify your model so that the harvesting rate is a parameter. Then use *dfield* to get an estimate of which values of the harvesting rate will ensure that all populations eventually die out.

   (f) For three different choices of the harvesting rate, estimate the minimum initial population required to produce a sustainable population. Only use harvesting rates for which it is possible to have a sustainable population.

More questions over page...

3. **Challenge question**: Consider the differential equation

$$\frac{dy}{dt} = ky - \frac{y}{1+y^2}$$

where k is a constant.

(a) Use *dfield* to plot the slope field and some solutions for this differential equation in the case that $k = -1$. Sketch the phase line.

(b) Use *dfield* to work out how solutions to the differential equation change as the parameter k is varied. You should concentrate on how the number and type of equilibrium solutions varies with k. **Hint:** Focus on k-values in the interval [-2,2] and note that the y value at equilibrium solutions can vary widely for k in this interval - so make sure the ranges in your *dfield* window are set large enough.