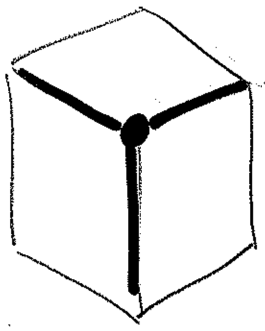
# Why there are only five Platonic solids

[Refer to Slide 1
(over the page) ]

Let Mysterydron be any solid equivalent by distortion to the sphere having :

- a constant number of edges per vertex, ev, say.

E.g.  ev = 3

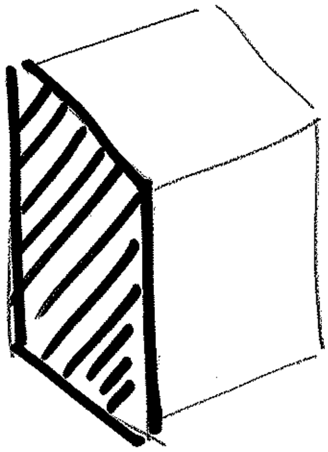
- a constant number of edges per face, ef, say.

$e_v$ $e_f$

| | Number of Vertices | Number of Edges | Number of Faces | $V - E + F$ | Edges per Vertex | Edges per Face |
|---|---|---|---|---|---|---|
| **Tetrahedron** | 4 | 6 | 4 | 2 | 3 | 3 |
| **Cube** | 8 | 12 | 6 | 2 | 3 | 4 |
| **Octahedron** | 6 | 12 | 8 | 2 | 4 | 3 |
| **Dodecahedron** | 20 | 30 | 12 | 2 | 3 | 5 |
| **Icosahedron** | 12 | 30 | 20 | 2 | 5 | 3 |
| Mysterydron | | E | | | | |

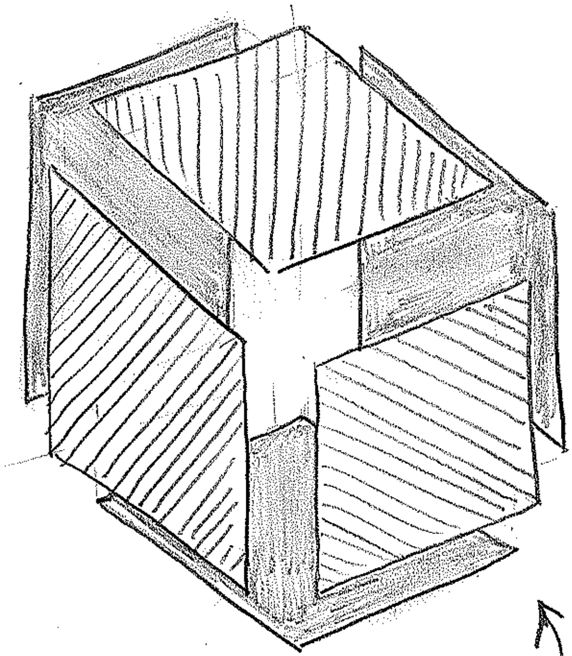
1'

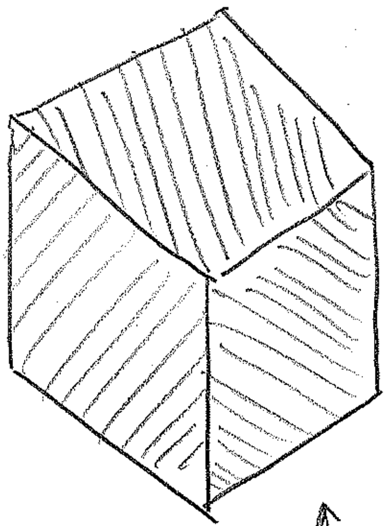
E.g.



$ef = 4.$

## Calculating E from F





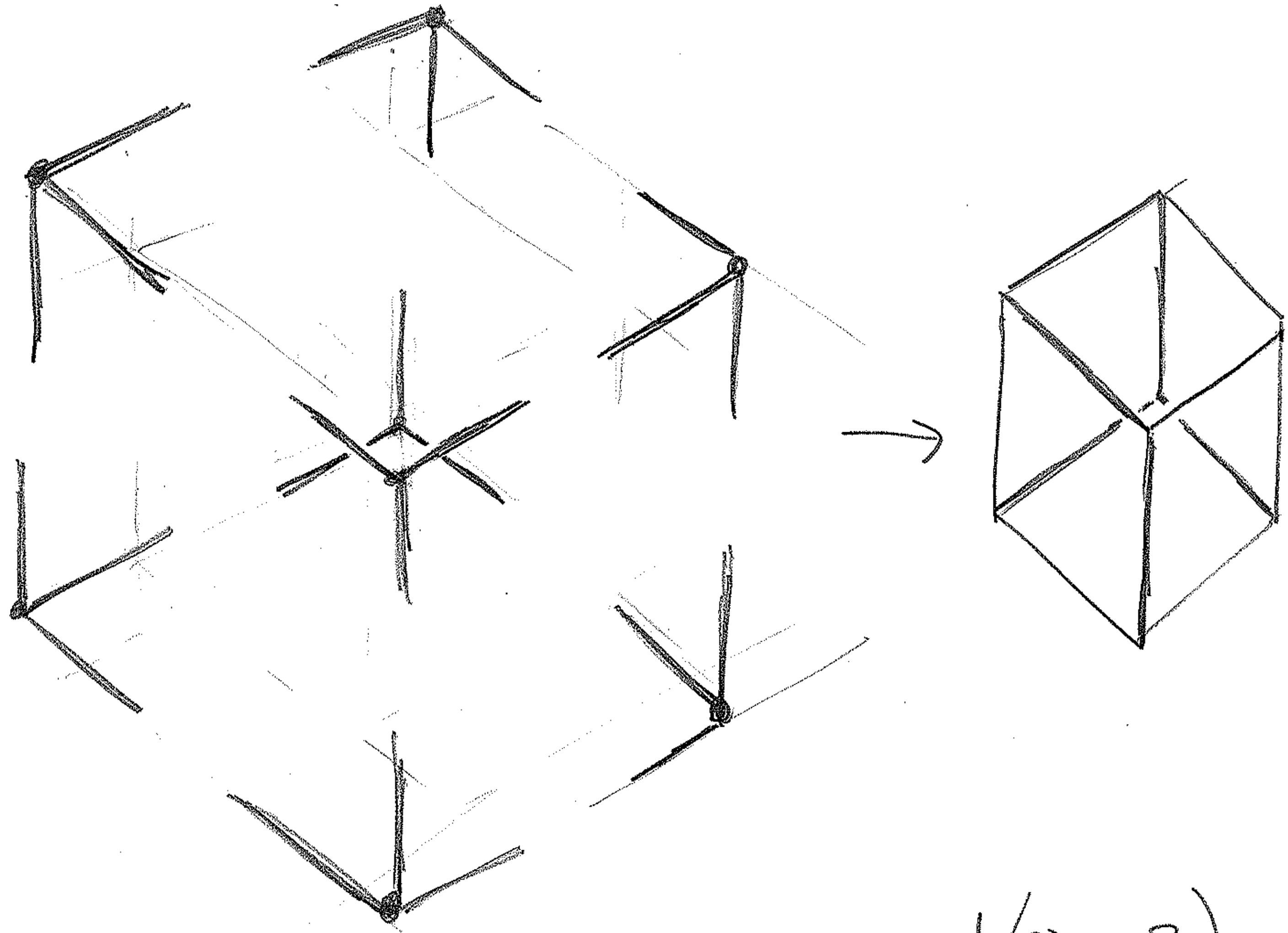
$E = 6 \times 4$

$E = \frac{1}{2}(6 \times 4)$

More generally : $E = \frac{1}{2}(F \times ef)$

$$\Rightarrow \boxed{F = \frac{2E}{ef}}$$

# Calculating E from V



$$E = \tfrac{1}{2}(8 \times 3)$$

$$E = 8 \times 3$$

More generally : $E = \frac{1}{2}(V \times ev)$

$$\Rightarrow \boxed{V = \frac{2E}{ev}}$$

## Apply Euler Characteristic Theorem:

$$V - E + F = 2$$

$$\Rightarrow \frac{2E}{ev} - E + \frac{2E}{ef} = 2$$

$$\boxed{\underbrace{\left(\frac{2}{ev} + \frac{2}{ef} - 1\right)}_{\text{Must be positive !!}} E = 2}$$

So $\frac{2}{ev} + \frac{2}{ef} > 1$

| $ev$ | $ef$ | $\frac{2}{ev} + \frac{2}{ef}$ | | |
|---|---|---|---|---|
| 3 | 3 | $\frac{2}{3} + \frac{2}{3} = \frac{4}{3} \geqslant 1$ | ✓ | tetrahedron |
| 3 | 4 | $\frac{2}{3} + \frac{1}{2} = \frac{7}{6} \geqslant 1$ | ✓ | cube |
| 4 | 3 | $\frac{1}{2} + \frac{2}{3} = \frac{7}{6} \geqslant 1$ | ✓ | tetrahedron |
| 4 | 4 | $\frac{1}{2} + \frac{1}{2} = 1$ | ✗ | |
| 3 | 5 | $\frac{2}{3} + \frac{2}{5} = \frac{16}{15} \geqslant 1$ | ✓ | dodecahedron |
| 5 | 3 | $\frac{2}{5} + \frac{2}{3} = \frac{16}{15} \geqslant 1$ | ✓ | isocahedron |
| 3 | 6 | $\frac{2}{3} + \frac{1}{3} = 1$ | ✗ | |
| 6 | 3 | $\frac{1}{3} + \frac{2}{3} = 1$ | ✗ | |

No more !!