Maths 190 Lecture 8

Topic for today:

More on Infinity

Question of the day:

How much bigger is the set of rational numbers than the set $\{1,2,3,\ldots\}$?

Recall from last lecture

Two sets are of the same size if and only if there is a one-to-one correspondence between members of one set and members of the other set.

Examples

- $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

  $\{2, 3, 4, 5, 6, 7, 8, 9, 10\}$

- $\{1, 2, 3, \ldots\}$

  $\{2, 3, 4, \ldots\}$

- $\{\ 1,\ \ 2,\ \ 3,\ \ 4,\ \ 5,\ \ 6,\ \ 7\}$

  $\{-3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3\}$

- $\{\ 1,\ \ 2,\ \ 3,\ \ 4,\ \ 5,\ \ 6,\ \ 7, \dots\}$

  $\{\dots, -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3, \dots\}$

One-to-one pairings are frequently shifts or shuffles.

The ping-pong ball conundrum

We do a thought experiment lasting exactly 60 seconds.

Start with a large empty barrel and a long line of ping-pong balls, numbered in order 1,2,3,. . . . Start the clock.

- In the first 30 seconds, put the first 10 balls into the barrel (numbers 1-10), find number 1 and throw it out.
- In half the remaining time (15 s), put the next 10 balls into the barrel (numbers 11-20), find number 2 and throw it out.
- In half the remaining time (7.5 s), put the next 10 balls into the barrel (numbers 21-30), find number 3 and throw it out.
- Continue in this way until 60 seconds has passed, then stop.

How many balls are now in the barrel?

Ping-pong ball variation

What is the outcome of the ping-pong ball experiment if the balls are not numbered? Discuss this in pairs.

(You could imagine that the balls are numbered with invisible ink that only the experimenter can see.)

Are there any sets bigger than $\{1, 2, 3, \ldots\}$?

Is $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$ bigger than $\{1, 2, 3, \ldots\}$?

Is the set of all rational numbers (fractions) bigger than $\{1, 2, 3, \ldots\}$?

Important ideas from today:

- Ideas about infinity are often counter-intuitive - intuition based on finite sets does not always work for infinite sets.
- The idea of one-to-one correspondence developed with finite sets does carry over to infinite sets and is our main tool for comparing the size of infinite sets.
- The set of all fractions (rational numbers) is the same size as the set $\{1, 2, 3, \ldots.\}$ (natural numbers).

For next time

- Read §3.2 in the textbook.
- Try some Mindscapes at the end of §3.2 of the textbook.
- Review the game of Dodgeball from Lecture 1 (Story 5 in section 1.1 of the textbook).
- Try to explain the ping-pong ball conundrum to a friend who is not in Maths 190.