THE UNIVERSITY OF AUCKLAND
NEW ZEALAND
Te Whare Wānanga o Tāmaki Makaurau

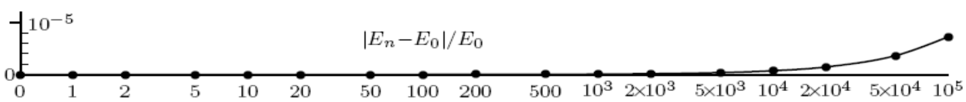
MATHS 787 – Special Topics course

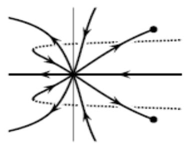
Numerical methods for differential equations
Semester 2 2007

This course is intended for students who are familiar with standard methods for solving ordinary differential equations, such as Runge–Kutta methods and linear multistep methods, or who have an interest in learning about these methods.
The content will be divided into three parts. First we will consolidate and formalise existing knowledge of the traditional methods. We will then introduce some new and more specialised topics, some of which are associated with so-called "General linear methods", which are generalisations of both Runge-Kutta and linear multistep methods. Finally, we will go more seriously into some of the new topics, with the actual selection based on interests that will have developed amongst members of the class.
Throughout the course the emphasis will be balanced between theoretical and practical considerations.

Taking account of these remarks, a list of topics, from which a selection will be made, is as follows:
- **Linear multistep methods: convergence, consistency, stability and order**
- **The first Dahlquist barrier on the order of convergent linear multistep methods**
- **Order conditions for Runge–Kutta methods**
- **Derivation of high order explicit Runge–Kutta methods**
- **Implicit Runge–Kutta methods**
- **A-stability barriers**
- **Implementation of implicit Runge–Kutta methods**
- **General linear methods: convergence, consistency, stability and order**
- **Order and stability barriers for general linear methods**
- **Construction and implementation of practical general linear methods**
- **Introduction to structure-preserving methods**

Timetable: Room 401, 4-6 Tuesdays and Thursdays, starting 24 July.
Note: no classes on 16-20 July.
The pictures below are relevant to topics arising in this subject.



The first picture represents **symplectic behaviour for Hamiltonian problems**. Even though small perturbations in the initial values of either position or the momentum are not preserved with time, the area of a set of possible initial perturbations is preserved. One of the aims for numerical methods for solving this type of problem is to obtain the same conservation in computed results.
The second picture shows **what happens when the energy of a certain mechanical problem is computed after many time steps**. The method does not exactly preserve this quantity but it keeps it quite close for a large number of steps.
The third picture shows lines drawn on a certain **Riemann surface which interrelates order and stability of a certain numerical approximation**. A consideration of pictures like this one can be used to decide when highly accurate methods can be used to obtain stable numerical results.
You are more welcome to ask for more information about any of these pictures, whether or not you are interested in actually enrolling in paper 787.

Further details can be obtained from the:
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