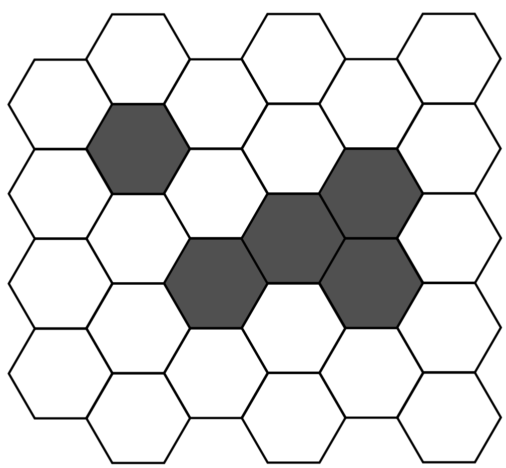
- Put your completed assignment in the appropriate box in the basement of the Mathematics/Physics Building **before** 4pm on the date due.
- Late assignments or assignments placed in the wrong box will not be marked.
- Your assignment **must** be accompanied by a blue Mathematics Department coversheet. Copies of the coversheet are available in the basement.
- **Tutorial write up:** Remember to hand in with your assignment your written solutions to the starred problems in Tutorial 9, 10 and 11. Each of these counts for **4 marks**.
- Each question in this assignment is worth **5 marks**.

1. The hexagonal Game of Life is a variant of the Game of Life, played on a hexagonal grid, so that each cell has six neighbours. The rules are the same, namely:

- A dead cell comes alive if it has exactly 3 live neighbours.
- A live cell remains alive if it has 2 or 3 live neighbours.
- A live cell dies if it has fewer than 2 or more than 3 live neighbours.

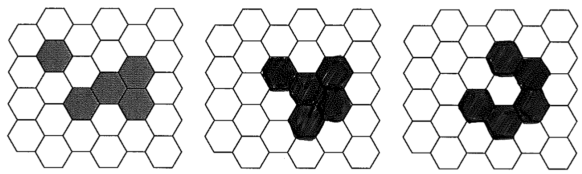
(a) Using these rules, find the next two generations from the following starting configuration (grey cells are alive, white cells are dead).
Note: Some hexagonal templates are attached to the end of the assignment for your answer.



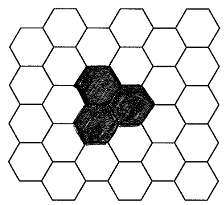
(b) Find a stable configuration with *at least* 4 live cells.

Solution:

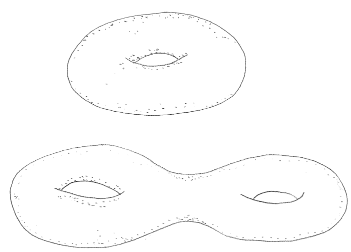
Part (a)



Part (b) There are many possible solutions. Here is one.

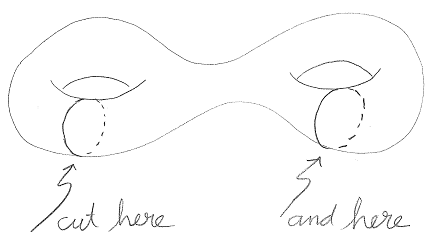


2. Provide a convincing reason why the torus and the two-holed torus are not equivalent by distortion.

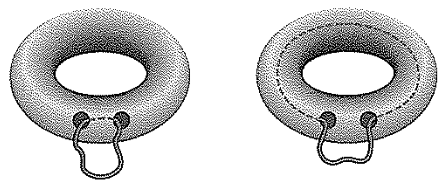


Solution: Here's one argument (among several). Any circular cut on the torus which does *not* separate it into two pieces turns it into a surface that can be distorted into a cylinder. A *circular* cut on a cylinder always divides it into two pieces. So, it follows that there is no way to perform two circular cuts on the torus without separating it into two (or more) pieces.

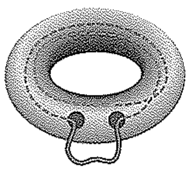
On the other hand, if one cuts the two-holed torus as shown below, then it stays in one piece. For these reasons, the torus and the two-holed torus cannot be equivalent.



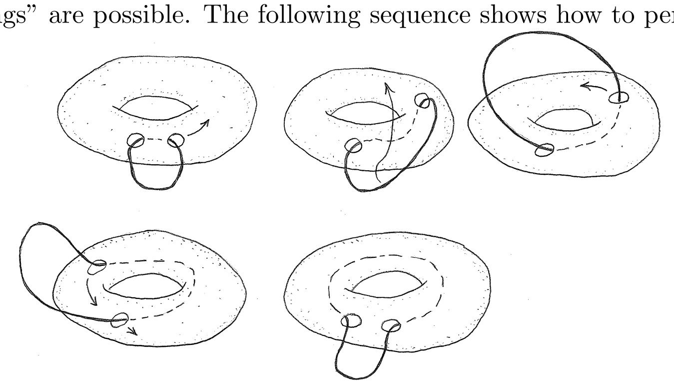
3.



34. Lasso that hole. Consider the two tori on the right. Both have two punctures on their sides. On the first torus, a rope is looped through the two holes but does not go around the hole of the torus. On the second, the rope is looped around the hole of the torus. Is it possible to distort the first torus to look like the second? How about if the rope looped around the hole twice, as shown on the left?

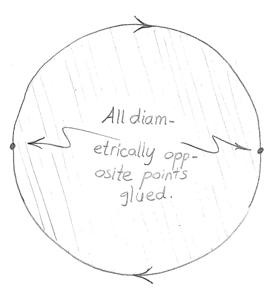


Solution: Both torus "loopings" are possible. The following sequence shows how to perform the first looping:



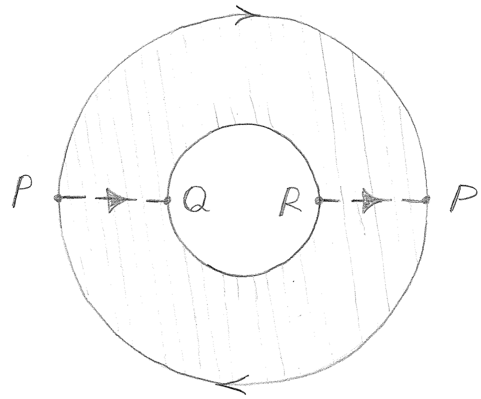
For the second looping, perform the first looping just described and then have the two holes orbit one-another clockwise through 180° — so the left hole moves to where the right hole is and vice-versa — and then repeat the movements described for the first looping once again.

4. In four-dimensional space it is possible to construct a single-sided surface, called the *projective plane*, by appropriately distorting a circular disk, and then gluing diametrically opposite points together:



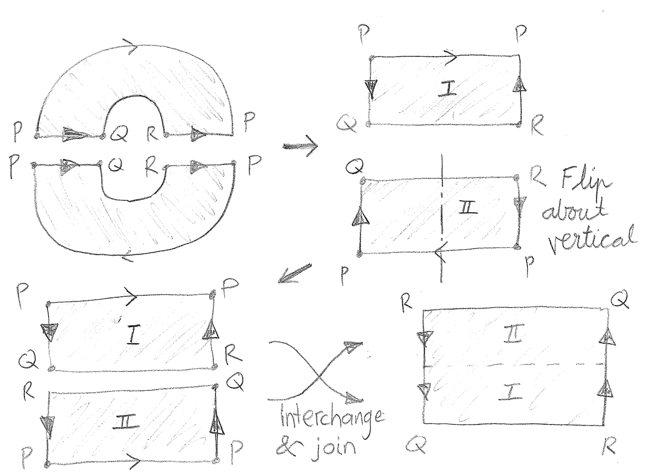
Show that the projective plane with a hole in it (a disk removed) is equivalent to the Möbius band.

Hint: Consider the following diagramatic representation of the projective plane with a hole in it:

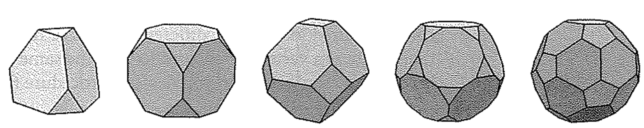


Now make an imaginary cut from point R to point Q. Note that the point P lies in the middle of this cut because of the outside gluing. Next show how to distort and rearrange the two halves obtained so that the outside diametric gluing creates a rectangle in which gluing back the RQ cut makes a Möbius band.

Solution: The last diagram in the following sequence of manipulations represents the Möbius band, as required.



5. The following collection of pictures shows the regular solids with their vertices cut off. Such objects are called *truncated solids*. For each truncated solid, count the number of vertices, edges, and faces, and verify that the Euler characteristic is 2 in every case.



Solution: Here is the computation of $V - E + F$ for each of the given solids: truncated tetrahedron, $12 - 18 + 8 = 2$; truncated cube, $24 - 36 + 14 = 2$; truncated octahedron, $24 - 36 + 14 = 2$; truncated dodecahedron, $36 - 66 + 32 = 2$; isocahedron, $60 - 90 + 32 = 2$.