# Maths 190 Lecture 12

**Topic for today**: Tiling the plane

**Question of the day**: What is the most symmetric shape that can be used to tile the plane?

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# Tilings of the plane

**In pairs or threes**: Use the handout and the transparencies provided to investigate the symmetries corresponding to each of the tiling patterns on the front page of the handout. Write a list of the symmetries for each case.

# Symmetry in tilings of the plane

We distinguish between two types of symmetries:

- A **rigid symmetry** of a pattern in the plane is a motion of the plane that preserves the pattern and does not shrink, stretch, or otherwise distort the pattern.

  Examples of rigid symmetries are shifts, rotations and flips.

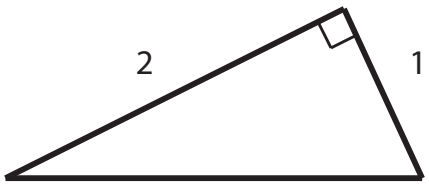
- A pattern in the plane has a **symmetry of scale** if the tiles that make up the pattern can be grouped into super-tiles that still cover the plane and, if scaled down, can be rigidly moved to coincide with the original pattern.

Note that these definitions refer to symmetries of a pattern, not to symmetries of the individual tiles.

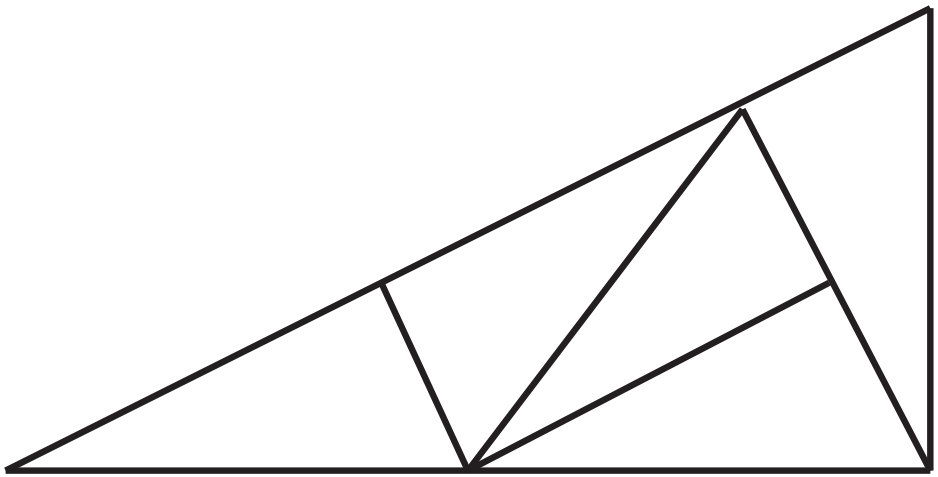
- Some tilings have rigid symmetries and symmetry of scale (e.g., tilings using square tiles or equilateral triangle tiles).
- Other tilings have rigid symmetries but no symmetry of scale (e.g., tilings using hexagonal tiles)
- Do any tilings have symmetry of scale but no rigid symmetries?

# The Pinwheel Pattern

The pinwheel pattern is a tiling of the plane using a triangular tile: a right angle triangle with sides of length 1, 2 and $\sqrt{5}$ units.

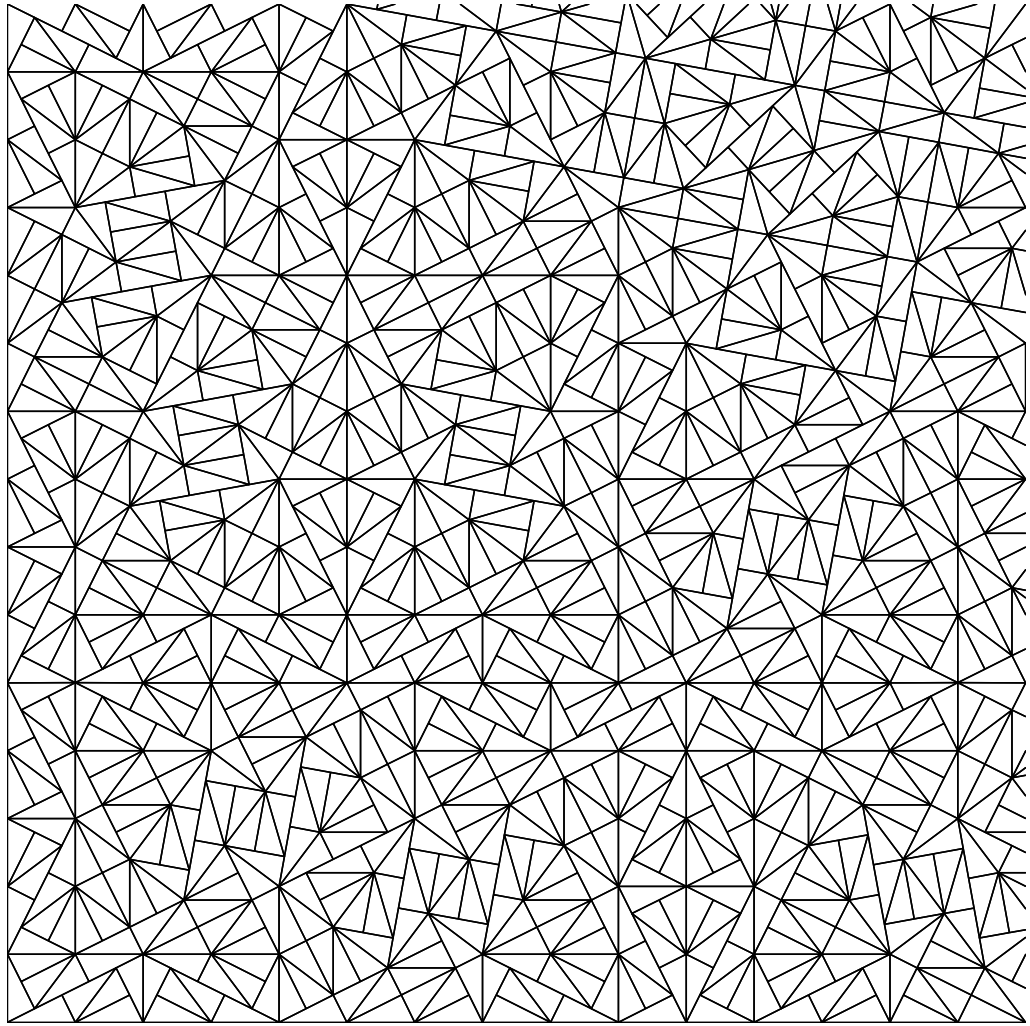



Five pinwheel triangles are combined to make one super-tile.



Five pinwheel super-tiles can be combined to make one super-super-tile, and so on . . .

The pinwheel pattern has symmetry of scale but no rigid symmetries. We say the pattern is **aperiodic**.



# Important ideas from today:

- Patterns in the plane can have two types of symmetries: rigid symmetries and symmetry of scale.
- Some patterns have symmetry of scale but no rigid symmetries. The pinwheel pattern is an example of this type of pattern.

# For next time

- Read §4.4 in the textbook.
- Try some Mindscapes at the end of §4.4 of textbook.
- Look about you as you go about your life in the next few days and notice the tiling patterns you see. What symmetries can you see?