Maths 190 Lecture 17

- **Topic for today:** The Dynamics of Change
- **Question of the day**: What do you need to know to predict the population of the world in the future?

Many physical systems have a “repeating” quality?

Examples from our world of things that repeat:

- Tides
- Motion of planets
- Rush hour

Iterative Dynamical Systems

A simple model of a bank balance

- Let B_n be the amount of money in a bank account at time n. If no withdrawals are made and the interest rate is 3% per year, then

  $$B_{n+1} = 1.03B_n$$

- If the balance is initially $B_0 =$\$5000, then

  $$B_1 =$$

  Balance after 10 years is

Iterative Dynamical Systems

A really simple population model

- Let P_n be the 'population density' at time n. A population increasing by a constant percentage each year could be modelled by:

$$P_{n+1} = kP_n$$

- If $k = 2$ and $P_0 = 0.1$, then

$$P_1 =$$

A slightly less simple model

- A slightly less simple population model:

$$P_{n+1} = k(P_n - P_n^2)$$

- $k = 2$, $P_0 = 0.1$:

$$P_1 =$$

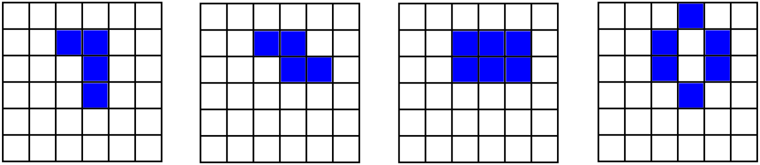
- $k = 4$, $P_0 = 0.1$:

$$P_1 =$$

- Often simple rules can have complicated behaviour.

Conway's Game of Life

The Game of Life

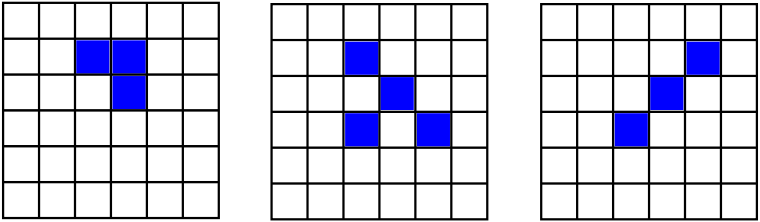


- The game is played on an infinite grid of ‘alive’ and ‘dead’ cells. Live cells are shaded, dead cells are blank.
- Each cell has eight “neighbours”. The current status of a cell’s neighbours determines the status (dead or alive) of the cell in the next generation.
- No random element in the game; just a few simple rules.

Rule 1: Staying alive

A living square will remain alive if it has two or three neighbours, otherwise it will die.

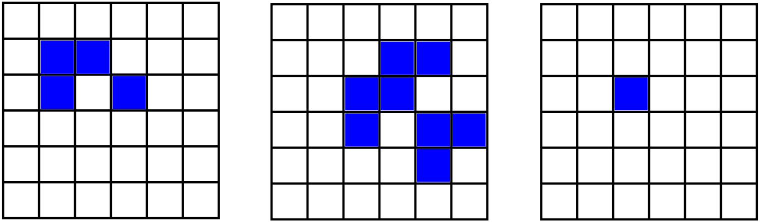
With the following initial configurations, which cells remain alive in the next generation?



Rule 2: Dying

A living square will die if it has fewer than two, or more than three neighbours.

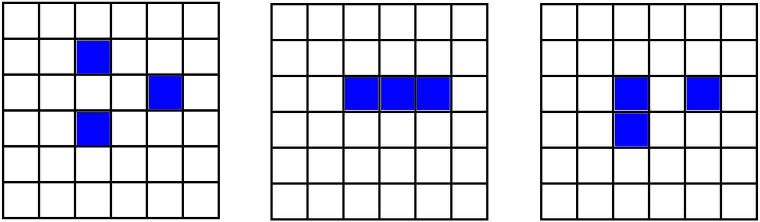
With the following initial configurations, which cells will die in the next generation?



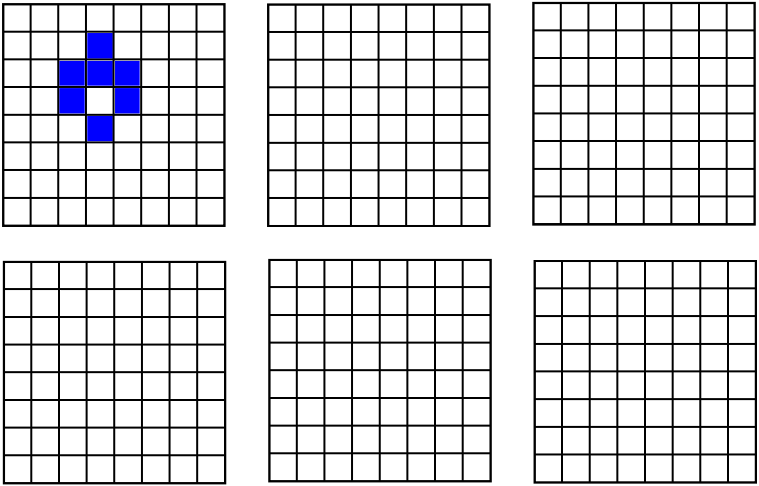
Rule 3: Coming alive

A dead square will come alive if it has *exactly* three living neighbours.

With the following initial configurations, which cells will come alive in the next generation?



Have a go!



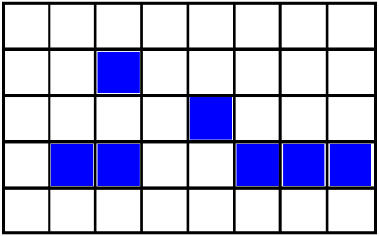
Types of behaviour

Types of behaviour we might see include:

- population explosion
- population extinction
- steady pattern
- periodic pattern
- migratory pattern

Types of behaviour

Sometimes the behaviour is very complicated. For instance, this initial configuration takes 5206 steps to settle down to a steady pattern.

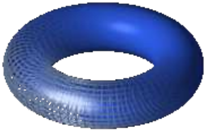


There's no way to tell from an initial configuration what will happen — just have to try it and see.

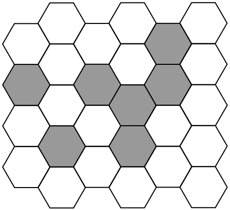
See `http://www.bitstorm.org/gameoflife/` for more information.

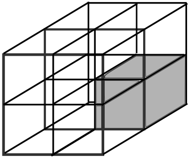
Other cellular automata

- Change the rules.
- Change the space.



- Change the grid.





Important ideas from today:

- The growth of populations can be modeled using fairly simple repeating rules.
- The Game of Life is one example of a simple population model.

For next time

- Read section 6.5