DEPARTMENT OF MATHEMATICS

MATHS 190 **Lecture 22 Summary**

Remember that a graph is a set of dots (vertices) with lines (edges) between them. Consider a connected graph which can be drawn on a sphere without any edges crossing. Let E be the number of edges, V the number of vertices and F the number of faces (since the graph is drawn on the surface of a sphere we must remember to count the "outside" face).

The **Euler characteristic** is the formula $V - E + F = 2$. You need to memorise this formula.

The proof of this formula relied on the fact that every connected graph can be built in the following way: Start with the graph having a single vertex and no edges; repeatedly add an edge between two existing vertices or add a new vertex and an edge between it and a previously existing vertex. This fact is not surprising; it is exactly how we draw graphs.

We then showed that if a graph satisfies $V - E + F = 2$ then the new graph, with an edge added in either of the above ways, also satisfies the formula.

We then considered a hypothetical platonic solid called MYSTERAHEDRON and noted that this would lead to a graph on the sphere. Let p be the number of edges of each face and q the number of faces which meet at each vertex. We deduced that $\frac{2}{q} + \frac{2}{p} > 1$. Hence, we found only the 5 platonic solids we already knew.

This lecture contained two big proofs. The first (proving the formula for the Euler characteristic) works by building up a complicated graph by a sequence of simple steps. Make sure you understand this proof, as you may be asked to write it down in the exam. The second proof (that there are only 5 platonic solids) is more algebraic, and you are not required to know this proof.

Before you come to the next lecture: Think about the two big proofs in this lecture. Make sure you understand the "structure" of the proofs and the logic behind them.

- Read §5.4 in the textbook.
- Try some of the Mindscapes at the end of §5.4 in the textbook.