

Maths 190 Assignment 1 Solutions

March 23, 2010

Due:

1. (3 marks) Profit was \$2. One possible explanation: think of the transactions as two separate deals, first she buys an item for \$7 and sells it for \$8 making \$1 profit. Then she buys another item for \$9 and sells it for \$10, making another \$1 profit. The fact that they were the same item is irrelevant. Also award marks for other sensible explanations.

2. (6 marks) My ordering is:
 - Distance in meters you will walk in your life:
 Assume 5000 steps per day, step length approx $1m$, so $5000m$ per day.
 Life expectancy approx 70 years is $70 \times 365 \approx 25,000$ days.
 So total is $5 \times 10^3 \times 2.5 \times 10^4 \approx 1.2 \times 10^8$.
 - The number of grains of rice that will fit in MLT3:
 Size of room is approx $15m \times 20m \times 4m = 1200m^3$
 Size of a grain of rice: cylinder approximately 7mm long, 1mm radius. One cubic metre could contain about $10^9 / (7 \times \pi \times 1^2) \approx 4.5 \times 10^7$ grains of rice. So total number is about $4.5 \times 10^7 \times 1.2 \times 10^3 \approx 5 \times 10^{10}$
 - Number of tennis balls you could fit inside the earth:
 Radius of the Earth is approx 6000km, so volume is approx $(4/3) \times \pi \times (6,000,000)^3 \approx 9 \times 10^{20}$.
 From lecture, approx 50 tennis balls in box which is approx $30cm \times 20cm \times 30cm = 0.18m^3$.
 So total number is $9 \times 10^{20} / 0.18 \times 50 \approx 2.5 \times 10^{23}$.

3. (a) (3 marks)
 - i. $210 = 2 \times 3 \times 5 \times 7$
 - ii. $3211 = 13 \times 13 \times 19$
 - iii. 4133 is prime.
 (b) (3 marks)
 - i. Counting the number of calculations: $210 \div 2 = 105$, $105 \div 2$ not an integer, $105 \div 3 = 35$, $35 \div 3$ not an integer, $35 \div 5 = 7$. 7 is less than 5^2 so can stop. Total is 5 calculations.
 - ii. $3211 \div \{2, 3, 5, 7, 11\}$ not an integer $3211 \div 13 = 247$, $247 \div 13 = 19$. 19 is less than 13^2 so can stop. Total is 7 calculations.
 - iii. Have to check division by all primes up to the square root, these are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, so have to do 18 calculations.

4. (a) (1 mark) 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610
 (b) (2 marks) 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364

(c) (3 marks) The sum $F_n + L_n$ is

2, 4, 6, 10, 16, 26, 42, 68, 110, 178, 288, 466, 754, 1220, 1974

which is twice the $(n + 1)$ th Fibonacci number. That is:

$$F_n + L_n = 2F_{n+1}$$

so

$$L_n = 2F_{n+1} - F_n.$$

5. (6 marks)

- (a) This must be true, by the pigeonhole principle. There are only 12 months, so if there were only 8 students with their birthdays in each month, that would only be $12 \times 8 = 96$ students. But there are 104 students, so there must be at least one month with at least 9 students.
- (b) Not necessarily true: there could be 9 students with birthdays in each month from January through August, and 8 students with birthdays in September through December, making a total of 104 students.
- (c) Not necessarily true: every student could have their birthday in January.
- (d) Not necessarily true: with only 104 students, this is fewer than 366 days, so each student could have their birthday on a different day.