

Discuss the following situations with one or two other students and try to work out together a solution to each question. Make sure everyone in the group contributes to the discussion. When you have an answer to a question that everyone agrees with, make some notes outlining how you worked out the answer. Then try to explain your answer to your tutor.

1. (Mindscape 30, 3.2 of text) A Double Decker package contains two delicious chocolate bars. Suppose you had infinitely many Double Decker packages: one for each natural number. Does the collection of individual chocolate bars have the same cardinality as the natural numbers? If not, explain why? If so, provide a one-to-one correspondence.
2. (Mindscape 32, 3.2 of text) Not-Finite City is made up of infinitely many roads running north and south (one road for each natural number) and infinitely many streets running east and west (one street for each natural number). A traffic light is placed at every intersection of a street with a road. How many traffic lights are there? Does the set of traffic lights have the same cardinality as the set of natural numbers? If not, explain why. If so, provide a one-to-one correspondence.
3. (Mindscape 4, 3.3 of text) Your friend gives you a list of three, five-digit numbers, but she only reveals one digit in each: $3????$, $?8???$, $??2??$. Can you write down a five-digit number you know for certain will not be on her list? If so, give one; if not, explain why not.
4. (Mindscape 12, 3.3 of text) Consider the following infinite collection of real numbers. First, describe in your own words how these numbers are constructed (that is, describe the procedure for generating this list of numbers). Then, using Cantor's diagonalization argument, find a number not on the list. Justify your answer.

0.123456789101112131415161718 ...

0.2468101214161820222426283032 ...

0.369121518212427303336394245 ...

0.4812162024283236404448525660 ...

and so on.

5.

Write up your answer to this question and hand it in with your answers to Assignment 2 (due August 26th). Don't forget to write down the names of the people in your tutorial group, so that you can acknowledge your collaborators in your report.

Prove that a circle of radius 1 cm has the same number of points as a square of side length 2 cm. Stated precisely, prove that the cardinality of points on the circle is the same as the cardinality of points on the square. Do this by describing a one-to-one correspondence between these two sets.

6. (Mindscape 13, 3.3 of text) Consider the infinite collection of boxes below:

$\square - \square - \square - \square - \square - \square - \square - \square - \square - \square - \square - \square - \square - \square - \square - \square - \dots$

Suppose you have two markers, one red and one blue, and you colour each box one of the two colours. How many different ways could you colour the collection of boxes? Show that the set of all possible box colourings has a greater cardinality than the set of all natural numbers.

7. (Mindscape 16, 3.5 of text) Prove that the cardinality of points in a solid cube is the same as the cardinality of points on a line segment.
8. This Mindscape is harder - only attempt it once you have good answers to the other tutorial questions.

(Mindscape 19, 3.3 of text) Recall the Hotel Cardinality from the last tutorial. It is a hotel with as many rooms as there are natural numbers. The room numbers are 1,2,3,4,5, ... Create a collection of people or groups of people so that it would be impossible for the night manager to give each person or group a room. Explain why it is not possible to give each person or group from the collection a room.