EXTRAPOLATION OF SYMMETRIZED RUNGE KUTTA METHODS

Presenter : Annie Gorgey, Supervisor: Robert Chan

Abstract

Symmetrization is a process of preserving the asymptotic error expansion of symmetric methods in even powers of the stepsize as well as providing damping for stiff problems. This can be achieved by a related Runge-Kutta (RK) method called a symmetrizer with the property that its stability function $R(z)$ satisfies $R(\infty) = 0$. This is an important property in the numerical solution of stiff problems because it helps to dampen errors. Symmetrization of low-order symmetric methods (midpoint and trapezoidal rules, linearly implicit methods), called smoothing, has been studied by Gragg [1], Bulirsch and Stoer[2] for nonstiff problems and by Lindberg [3], Dahlquist [4] and Deuflhard [5] for stiff problems. Chan [6] generalized the concept of smoothing to arbitrary symmetric RK methods. In this study we construct symmetrizers for Gauss methods with 2 and 3 stages and examine the error behaviour, both theoretically and experimentally, for the Prothero-Robinson problem in particular. We have observed that extrapolation of the implicit midpoint rule or the implicit trapezoidal rule with a symmetrizer shows much better behaviour than extrapolation without a symmetrizer. This study shows that the observation is also true of extrapolation with higher order symmetric methods. We compare extrapolation applied in two different modes (active and passive). It is not yet clear from our study which mode is more reliable and accurate.

References

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