FORBIDDEN MINORS AND GRAPH VISTAS

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ABSTRACT. The graph minor theorem has given graph theorists a new way of looking at graphs. We introduce the result and discuss the direction we are taking with it.

Kuratowski's theorem characterises planar graphs as those which do not have either K_5 or $K_{3,3}$ as a graph minor. Wagner's Conjecture is a far-reaching generalisation of Kuratowski's theorem, proposing that any graph property closed under the graph minor operations of vertex deletion, edge deletion and edge contraction can be characterised by a *finite* set of forbidden minors. Robertson and Seymour proved this in a series of papers over 20 years, finishing in 2004. We discuss the main result, which hinges on the well-quasi-ordering of the class of finite graphs.

Much research in this field has since focused on finding obstruction sets to particular properties, but we turn the question around. Given a set of forbidden minors, we investigate the property which the set obstructs. We have focused on operations on graphs in the obstruction set and how these affect the property. Three operations reverse the standard graph minor operations: starting with a single graph G, we construct a set of graphs, each element of which produces G under one of the graph minor operations. Future work will look at other operations on sets of forbidden minors such as unions and intersections.

References

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