Department of Mathematics

Maths 190 Assignment 2 Solutions

April 27, 2010 Due:

1. (4 marks, 1 for each)

   (a) Required computation is:

   $$0\times 1+3\times 2+8\times 3+7\times 4+9\times 5+5\times 6+3\times 7+6\times 8+4\times 9+7\times 10 \mod 11$$

   which simplifies to

   $$0+6+2+6-10-3-1+4-8-7=-11\equiv 0 \mod 11$$

   so the ISBN is correct.

   (b) Required computation is:

   $$0\times 1+6\times 2+9\times 3+1\times 4+1\times 5+3\times 6+1\times 7+1\times 8+8\times 9+4\times 10 \mod 11$$

   which simplifies to

   $$0+1-6+4+5+7+7-3-16-4=-5\neq 0 \mod 11$$

   so the ISBN is incorrect.

   (c) Required computation is:

   $$0\times 1+2\times 2+0\times 3+1\times 4+2\times 5+5\times 6+9\times 7+8\times 8+3\times 9+1\times 10 \mod 11$$

   which simplifies to

   $$0+4+0+4-1-3-14-2-6+10=-8\neq 0 \mod 11$$

   so the ISBN is incorrect.

   (d) Required computation is:

   $$0\times 1+4\times 2+9\times 3+5\times 4+0\times 5+1\times 6+2\times 7+6\times 8+5\times 9+3\times 10 \mod 11$$

   which simplifies to

   $$0+8-6-2+0+6+3+4-10-3=0 \mod 11$$

   so the ISBN is correct.

2. (a) (1 mark) $\sqrt{4}=2$ which is rational.

   (b) (3 marks) The mistake is in line *ii*. Just because a^2 is a multiple of 4 does not mean that a is a multiple of 4. For instance, 36 is a multiple of 4, but 6 is not.

3. (a) (1 mark) For example, 1.25995.

   (b) (1 mark) For example, 1.25993.

   (c) (3 marks). Yes, it is always possible. Write both numbers a and b as their decimal expansions, for simplicity we assume both a and b are between 0 and 1. Suppose the decimal expansions are the same up to the nth decimal place, but different in the $n+1$th place, then we can write

$$a = a_1 a_2 a_3 \dots a_n a_{n+1} a_{n+2} \dots$$
$$b = a_1 a_2 a_3 \dots a_n b_{n+1} b_{n+2} \dots$$

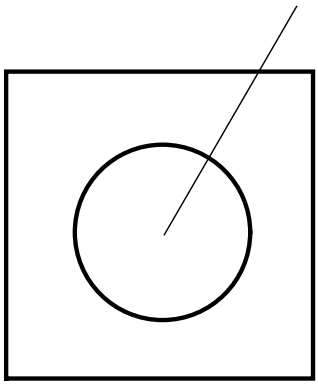
   Since $b > 1$, then $b_{n+1} > a_{n+1}$. It is clear that we can choose a number with a decimal expansion which is between a and b, and make it terminate, so that it is a rational number.

4. (3 marks each)

   (a) Set M has cardinality less than the cardinality of N. Since New Zealand has finite area, and each mobile phone has a minimum size (i.e., greater than one cubic cm, for instance), then the maximum number of phones that can exist in New Zealand is the total area, divided by the minimum size, which is still finite, so M is finite. [Accept similar arguments]

   (b) The cardinality of C is greater than the cardinality of N. The following picture shows that the cardinality of C is the same as that of the set of points in the interval $[0, 1]$, which is greater than that of N.

   We take the line segment $[0, 1]$ and wrap it into a circle. Putting the circle inside the square we can construct a one-to-one correspondence between the points on the circle and the points on the square, by picking a point inside the circle and drawing straight lines radially outwards. The point the line crosses the circle corresponds to the point the line crosses the square.



   (c) Set O has the same cardinality of N, since we can find a one-to-one mapping, as follows. Let 1 correspond with $0.10000\dots$, 2 correspond with $0.11000\dots$, 3 correspond with $0.111000\dots$, and so on, so the number n from N corresponds with the number in O which has n 1's followed by 0's.

   (d) The cardinality of F is larger than the cardinality of N. We use Cantor's diagonal argument to prove this. First suppose that the cardinality of F and N are the same, which means we can write a list of the elements of F. We then construct the number f as follows. If the nth digit of the nth element in the list is a 4, then the nth digit

of f is a 5. And conversely, if the nth digit of the nth element in the list is a 5, then the nth digit of f is a 4. Thus f differs from every element of F which is in the list in at least one position, but f is in F because it is only made up of 4's and 5's. This is a contradiction, so therefore F has greater cardinality than N.