

Maths 260 Assignment 4

October 5, 2010

Due: 4pm, Thursday October 14th, 2010

- Put your completed assignment in the appropriate box on the ground floor of the Maths/Physics building **before** 4pm on the date due. Late assignments or assignments placed in the wrong box will not be marked.
- Your assignment must be accompanied by a blue Mathematics Department coversheet. Copies of the coversheet are available from a box in the basement.
- You must show all your working.

1. Consider the system of differential equations

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 1 & -2 \\ 2 & a \end{pmatrix} \mathbf{Y}$$

where

$$\mathbf{Y} = \begin{pmatrix} x \\ y \end{pmatrix}$$

and a is a parameter.

- (a) For the case $a = -4$
 - i. Find the general solution to the differential equation. Express your answer in terms of real-valued functions.
 - ii. Sketch the corresponding phase portrait.
 - (b) Repeat (a) for the case $a = -3$.
 - (c) Repeat (a) for the case $a = -1$.
 - (d) For what values of a is the equilibrium at the origin a source?
2. The following equations model the growth of two populations living in a reserve:

$$\begin{aligned} \frac{dx}{dt} &= 3x - 4x^2 + 2xy, \\ \frac{dy}{dt} &= 10y - y^2 - 3xy, \quad x \geq 0, y \geq 0, \end{aligned}$$

where x and y are measured in hundreds of animals (i.e., $x = 1$ means there are one hundred animals of type x) and t is measured in years.

- (a) Explain briefly the physical significance of each term in the model.
- (b) Based on the model, what types of animals do you think might be modelled with this system?
- (c) Use *pplane* to draw the phase portrait for this system of equations. Print out your phase portrait and hand it in with your assignment.

- (d) Using the phase portrait from *pplane*, describe the long term behaviour of the two populations for all choices of initial population sizes.
- (e) It is decided to try to reduce the size of population x by removing 400 animals of type x from the reserve in each year. Write down a modified model that includes this effect.
- (f) Use *pplane* to draw the phase portrait for your modified model. Print out your phase portrait and hand it in with your assignment.
- (g) (Harder) Briefly discuss whether there are any qualitative changes in the behaviour of solutions in the modified model compared with the original model. Explain your answer carefully.

3. This question is about the following system of equations:

$$\begin{aligned}\dot{x} &= (2 - x)(y - 1) \\ \dot{y} &= x^2 - y\end{aligned}$$

A grid is provided at the back of this assignment. Use the grid for your answer to parts (c) and (d) of this question.

- (a) Find all equilibrium solutions and determine their type (e.g., spiral source, saddle).
- (b) For each of the equilibria you found in part (a), draw a phase portrait showing the behaviour of solutions near that equilibrium.
- (c) Find the nullclines for the system and sketch them on the answer sheet provided. Show the direction of the vector field in the regions between the nullclines and on the nullclines themselves.
- (d) Sketch the phase portrait of the system. Your phase portrait should show the behaviour of solutions near the equilibria, and should show a representative selection of solution curves *including* those passing through the following initial conditions:
 - i. $(x(0), y(0)) = (0, 1)$
 - ii. $(x(0), y(0)) = (3, 0)$

Make sure you show clearly where all your solution curves go in forward and backward time.

Grid for answering Question 3

