Maths 260 Lecture 6

- **Topics for today:**
  Modelling with DEs
- **Reading for this lecture:** BDH Section 1.1 again
- **Suggested exercises:** BDH Section 1.1, #15, 17
- **Reading for next lecture:** BDH Section 1.5
- **Today's handouts:** None

Modelling basics:

Using knowledge about how a quantity changes to write down a DE is called **modelling**, and a DE is a **model.**

The goal of modelling is to use the model to predict future values of the quantity being modelled.

Important steps in making a model:

1. Identify assumptions on which the model is based.
2. Identify all relevant quantities in the model.
3. Use assumptions in (1) to write down equations relating the quantities in (2).

We then use analytical, qualitative or numerical methods to find out about the behaviour of solutions to the model, and hence make predictions about the system being modelled.

Example 1: Making chilli

Mike is having a dinner party for a large group of people, and decides to make 8 litres of chilli.

The recipe calls for 2 teaspoons of hot sauce per litre of chilli, but Mike mistakenly puts in 2 tablespoons of hot sauce per litre (2 tablespoons = 6 teaspoons).

Mike doesn't want to throw the chilli out so he serves it anyway. However, as each person takes some chilli, Mike fills up the pot with beans and tomatoes without hot sauce until the concentration of hot sauce agrees with the recipe.

If the guests take one cup of chilli per minute from the pot (4 cups = 1 litre), how long will it take to get the chilli back to the recipe's concentration of hot sauce? How many cups of chilli will have been taken from the pot?

Example 2: A population model

A small population of animals is living in a large game park. Initially the population grows at the rate of 5% per year. The maximum population that can be supported by the resources of the park is 80,000.

(a) Write down a differential equation to model the way in which the population changes.

(b) It is decided to take 500 animals from the park every year. Modify your model to include a term or terms that model this.

(c) Use the model to determine the eventual size of the population if the initial population is

  (i) $13,000$

  (ii) $10,000$.

Example 3: Model of loan repayments

Sarah has a student loan of $20,000 when she completes her degree.

For the next two years Sarah makes no repayments and the loan accumulates interest at 6.8% per year. Thereafter, she pays off $3,000 per year and the interest rate remains at 6.8%.

When will Sarah finish paying off the loan?

Important ideas from today:

- When modelling a physical phenomenon (e.g., growth of a population) it is often easier to write down equations that describe the changes in the system rather than equations that describe the actual state of the system. This gives us a DE model.
- Qualitative, analytic and numerical methods may all give information about the behaviour of solutions to a particular DE model — the trick is to pick appropriate methods for the model being studied. Methods appropriate for one model may not be appropriate for a different model.