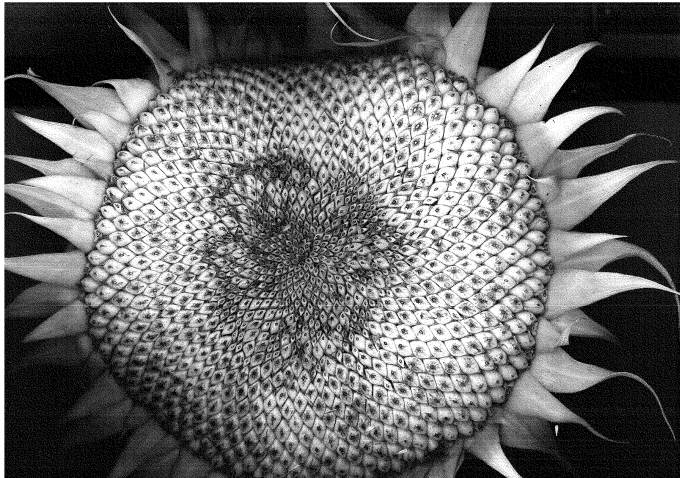▶ **Topic for today: Numerical Patterns in Nature**

▶ **Vitally important question**:

What do rabbits, pineapples, daisys, sunflowers and pinecones have in common?

Pineapples and pinecones

- ▶ Look carefully at your pineapple.
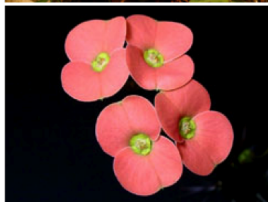- ▶ How many spirals does your pineapple have?

How many spirals in a sunflower?

How many spirals in a pinecone?

How many petals on a flower?

Is there a pattern?

- Put the numbers we have found in order.
-

- Can you spot a pattern?

Fibonacci Numbers

- $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots$
- Call these $F(1)$, $F(2)$, $F(3)$, etc
- Then
$$F(n) + F(n+1) = F(n+2)$$

She loves me, she loves me not...?

- Pulling petals off a flower, we'd like to know our chances of getting an odd number - "She loves me"!
- Most flowers have a Fibonacci number of petals.
- How many of the Fibonacci numbers are odd?

Which are odd?

- $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, \ldots$
- Looks like every third one is even.
- How can we show this is always true?

What about rabbits?

- ▶ Suppose we start with one breeding pair of rabbits.
- ▶ Each pair of rabbits produces one pair of offspring each month.
- ▶ It takes the rabbits one month to mature.
- ▶ How many rabbits do we have?

How many rabbits?

Ratio of $F(n)$

- What is the ratio $F(n+1)/F(n)$ as n gets large?
- Seems to converge...
- The Golden Ratio ψ.

$$\frac{1}{1} = 1$$

$$\frac{2}{1} = 2$$

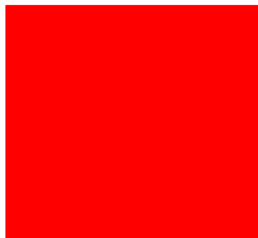$$\frac{3}{2} = 1.5$$

$$\frac{5}{3} = 1.6667$$

$$\frac{8}{5} = 1.6$$

$$\frac{13}{8} =$$

$$\frac{21}{13} =$$

$$\frac{34}{21} =$$

Which is your favourite rectangle?

Fibonacci Building Blocks

- ▶ Claim: Every natural number is either a Fibonacci number, or can be written as the sum of Fibonacci numbers.
- ▶ Try it!

- ▶ In fact, every natural number can be written as the sum of *non-consecutive* Fibonacci numbers.
- ▶ Use a "greedy" algorithm:
  - ▶ subtract the biggest possible Fibonacci number to get a smaller number.
  - ▶ repeat as required.

Important ideas from today:

- Certain patterns, and sequences of number reappear throughout nature. One common one is the Fibonacci sequence.
- Rabbits breeding, the numbers of spirals on pinecones and daisies, all follow the Fibonacci sequence.

For next time

- Read 2.3.