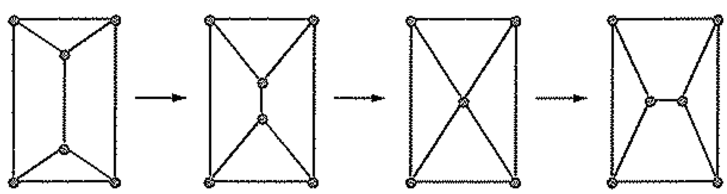
Tutorials in Maths 190 are **collaborative** tutorials. You should work in groups of 3 or 4 students, discussing the situations and puzzles listed below, or issues arising from lectures. Part of your final mark depends on your participation in tutorials.

Write up your answer to the question marked with a "⋆" and hand it in with Assignment 4 (due October 21st).

1. Regard the universal peace symbol as a graph in the plane and compute its Euler characteristic, $V - E + F$.

2. **Soap films.** Consider the following sequence of graphs generated by soap bubble films. As the vertical film shrinks, it passes through the unstable position in the third picture and then jumps to the last. Compute $V - E + F$ at each stage. What are the changes in the V, E, and F counts as we move from the second to the third picture?

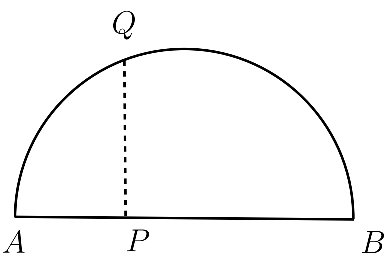


3. ⋆ Is it possible to draw a **connected** graph in the plane with an odd number of regions, an even number of vertices, and an even number of edges? (Don't forget to count the infinite "outside" of your graph as a region.) If so, draw one; if not, explain why not.

4. Is it possible to draw a **two-component** graph in the plane with an odd number of regions, an even number of vertices, and an even number of edges? If so, draw one; if not, explain why not.

5. Draw a connected graph on the torus that has Euler Characteristic 2. Do all connected graphs on the torus have Euler Characteristic 2? Explain.

6. Instead of the heated loop discussed in class, consider a heated semicircular wire:



Is the following "Hot Semicircle Conjecture" true?

> *There exists a point P on the bottom straight line, not equal to A or B, such that the point Q on the curved arc directly above has the same temperature as P.*

Explain your answer.

7. A hiker decides to climb up Mount Ruapehu. There is only one trail to the top. She starts at the base of the mountain at 8:00 a.m., Saturday. She climbs, stops, rests, backtracks a bit, but finally gets to the summit by 5:00 p.m. that evening. The next morning at 8:00 a.m. she begins to hike down. Again she stops, rests, backtracks (in fact, returns to the top because she left her tent there), but finally reaches the bottom (at the point where she started) by 5:00 p.m. that evening.

Must there exist a precise point (altitude) on the trail with the property that, at the very moment she crossed that point on Sunday, the hiker's watch showed precisely the same time it did on crossing that point the previous day?

8. An oceanographer conjectures that, at any moment in time, there are two points in the ocean, precisely fifty feet apart, having identical temperatures and salinity. Is she right? Why, or why not?