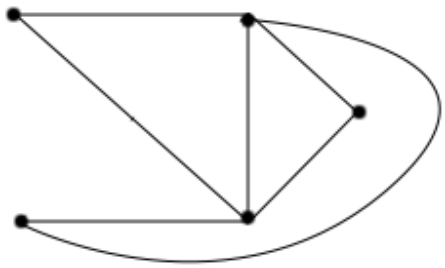
DEPARTMENT OF MATHEMATICS

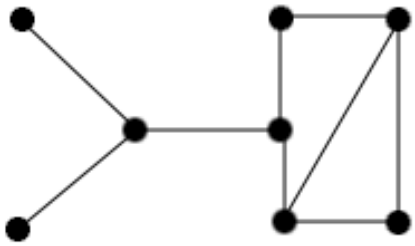
MATHS 190 Tutorial 11

Discuss the following situations with two or three other students and try to work out together a solution to each question. Make sure everyone in the group contributes to the discussion. When you have an answer to a question that everyone agrees with, make some notes outlining how you worked out the answer. Then try to explain your answer to your tutor.

1. Consider the following graph. How many vertices and edges does it have? Does the graph have an Euler circuit (i.e., a path starting from a vertex which returns to the vertex and passes along every edge exactly once)? If so, draw it.



2. Explain why the below graph does not have an Euler circuit. What is the minimum number of edges to be added to get a graph with an Euler circuit?



3. Is it possible to draw a connected graph with 4 vertices and 6 edges which does not have an Euler circuit?

4. Consider the graph in question 1 again. How many faces does it have? Verify the Euler formula $V - E + F = 2$.

5. **Write up your answer to this question and hand it in with your answers to Assignment 4 (due Monday May 31st).** See below for instructions on writing tutorial reports. Don't forget to write down the names of the people in your tutorial group, so that you can acknowledge your collaborators in your report.

   Is it possible to draw a connected graph in the plane with an odd number of faces, an even number of vertices, and an even number of edges? (Don't forget to count the infinite outside of your graph as a face.) If so, draw one; if not, explain why not.

6. Suppose G is a graph which is **not** connected and let V, E and F be the number of vertices, edges and faces. What are the possible values for $V - E + F$ which can occur? Can you describe a way to write down a graph which attains any one of these values?

7. A Hamilton circuit (this is not named after the bridges in the New Zealand town!) in a graph is a path of distinct edges which starts at a vertex, returns to the same vertex, and passes through every other vertex exactly once. (It does not need to cross every edge.)

   Do the graphs in Questions 1 and 2 have Hamilton circuits?

   An interesting fact is that there is a very simple criterion which explains exactly when an Euler circuit exists, but there is no known simple criterion for Hamilton circuits.

Writing up tutorial questions

A written solution for question 5 should be handed in for marking with Assignment 4. In your solution, you should include:

- the names of the people you discussed this with in your tutorial group;
- a clear statement of your solution to the puzzle;
- a clear explanation (in one or two paragraphs) or how you arrived at this solution; and,
- a statement of any assumptions you had to make in obtaining your answer.