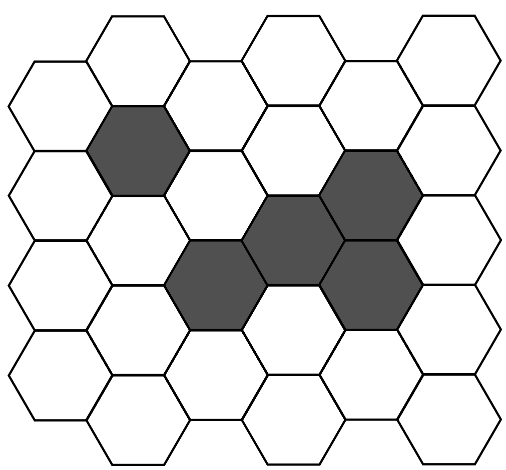
- Put your completed assignment in the appropriate box in the basement of the Mathematics/Physics Building **before** 4pm on the date due.
- Late assignments or assignments placed in the wrong box will not be marked.
- Your assignment **must** be accompanied by a blue Mathematics Department coversheet. Copies of the coversheet are available in the basement.
- **Tutorial write up:** Remember to hand in with your assignment your written solutions to the starred problems in Tutorial 9, 10 and 11. Each of these counts for **4 marks**.
- Each question in this assignment is worth **5 marks**.

1. The hexagonal Game of Life is a variant of the Game of Life, played on a hexagonal grid, so that each cell has six neighbours. The rules are the same, namely:

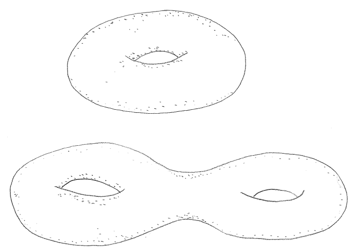
- A dead cell comes alive if it has exactly 3 live neighbours.
- A live cell remains alive if it has 2 or 3 live neighbours.
- A live cell dies if it has fewer than 2 or more than 3 live neighbours.

(a) Using these rules, find the next two generations from the following starting configuration (grey cells are alive, white cells are dead).
Note: Some hexagonal templates are attached to the end of the assignment for your answer.

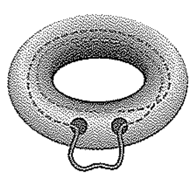


(b) Find a stable configuration with *at least* 4 live cells.

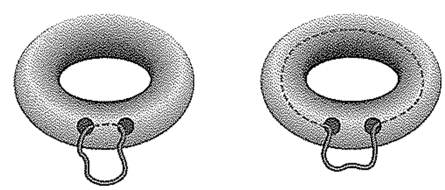
2. Provide a convincing reason why the torus and the two-holed torus are not equivalent by distortion.



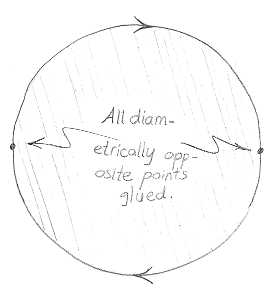
3.



34. Lasso that hole. Consider the two tori on the right. Both have two punctures on their sides. On the first torus, a rope is looped through the two holes but does not go around the hole of the torus. On the second, the rope is looped around the hole of the torus. Is it possible to distort the first torus to look like the second? How about if the rope looped around the hole twice, as shown on the left?

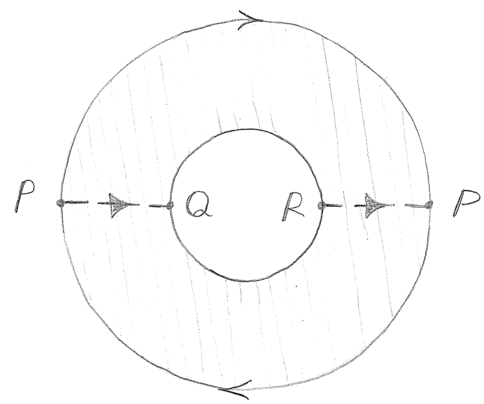


4. In four-dimensional space it is possible to construct a single-sided surface, called the *projective plane*, by appropriately distorting a circular disk, and then gluing diametrically opposite points together:



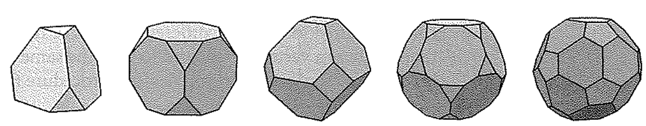
Show that the projective plane with a hole in it (a disk removed) is equivalent to the Möbius band.

Hint: Consider the following diagramatic representation of the projective plane with a hole in it:

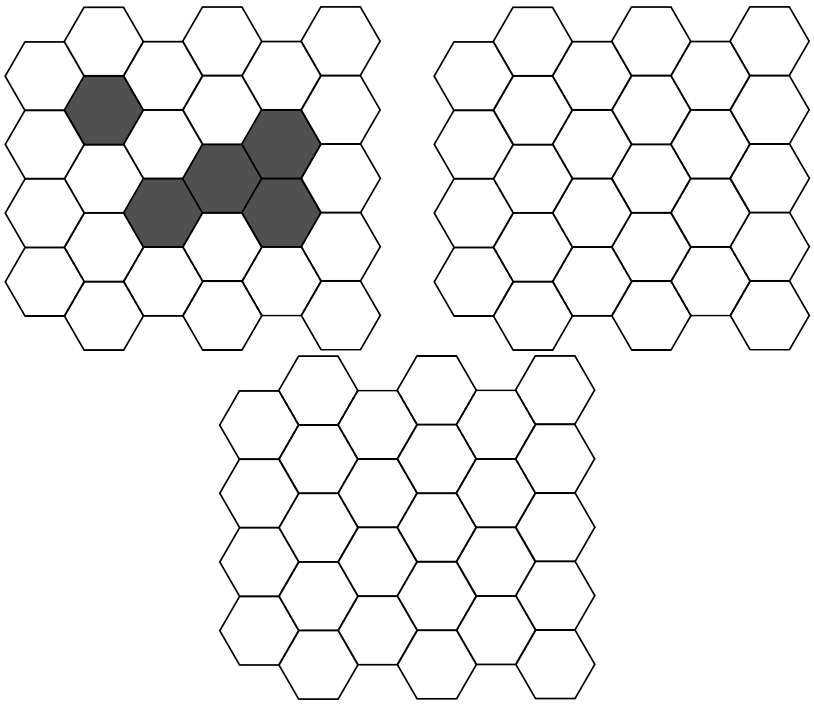


Now make an imaginary cut from point R to point Q. Note that the point P lies in the middle of this cut because of the outside gluing. Next show how to distort and rearrange the two halves obtained so that the outside diametric gluing creates a rectangle in which gluing back the RQ cut makes a Möbius band.

5. The following collection of pictures shows the regular solids with their vertices cut off. Such objects are called *truncated solids*. For each truncated solid, count the number of vertices, edges, and faces, and verify that the Euler characteristic is 2 in every case.



Template for Game of Life question, part (a):



Template for Game of Life question, part (b):