Maths 190 Lecture 15

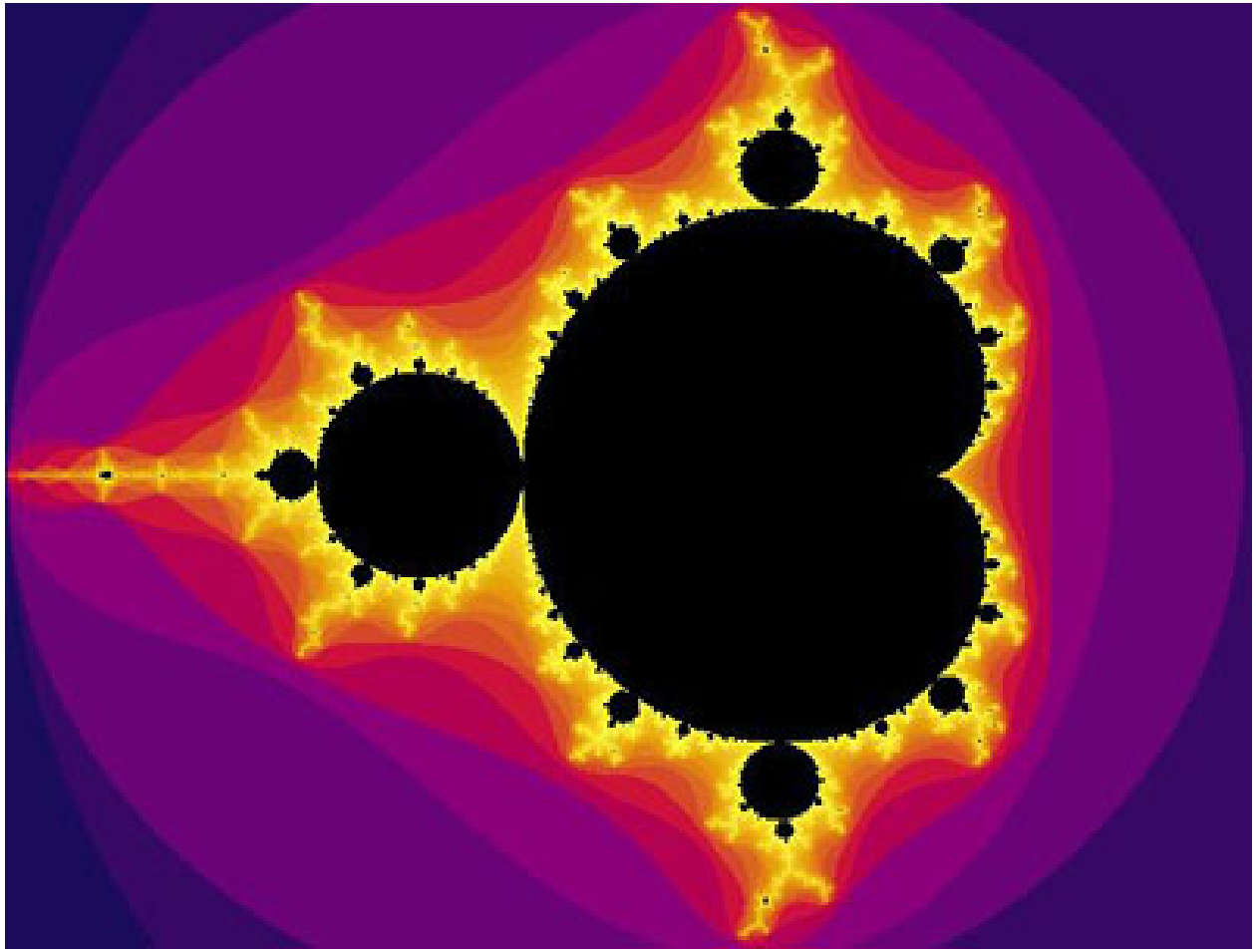
Topic for today: Self-similarity and fractals

First question of the day: How long is the coastline of NZ?

Second question of the day: How much broccoli is enough?

Fractals in nature

Mandelbrot set



How long is the coastline of New Zealand?

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What is a fractal?

There is no formal mathematical definition of a fractal. Informally, we say any image or structure with infinite detail is a fractal.

Building a self-similar fractal

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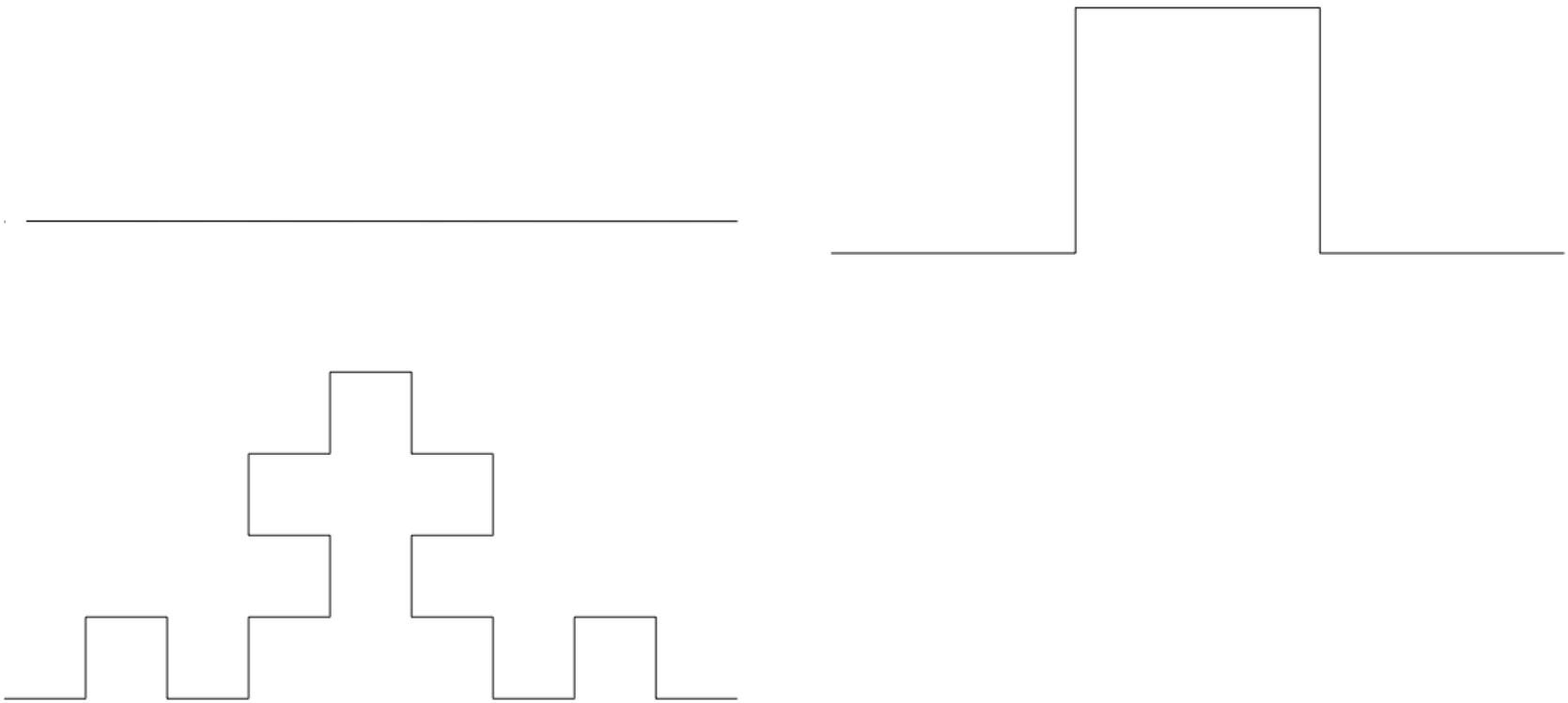
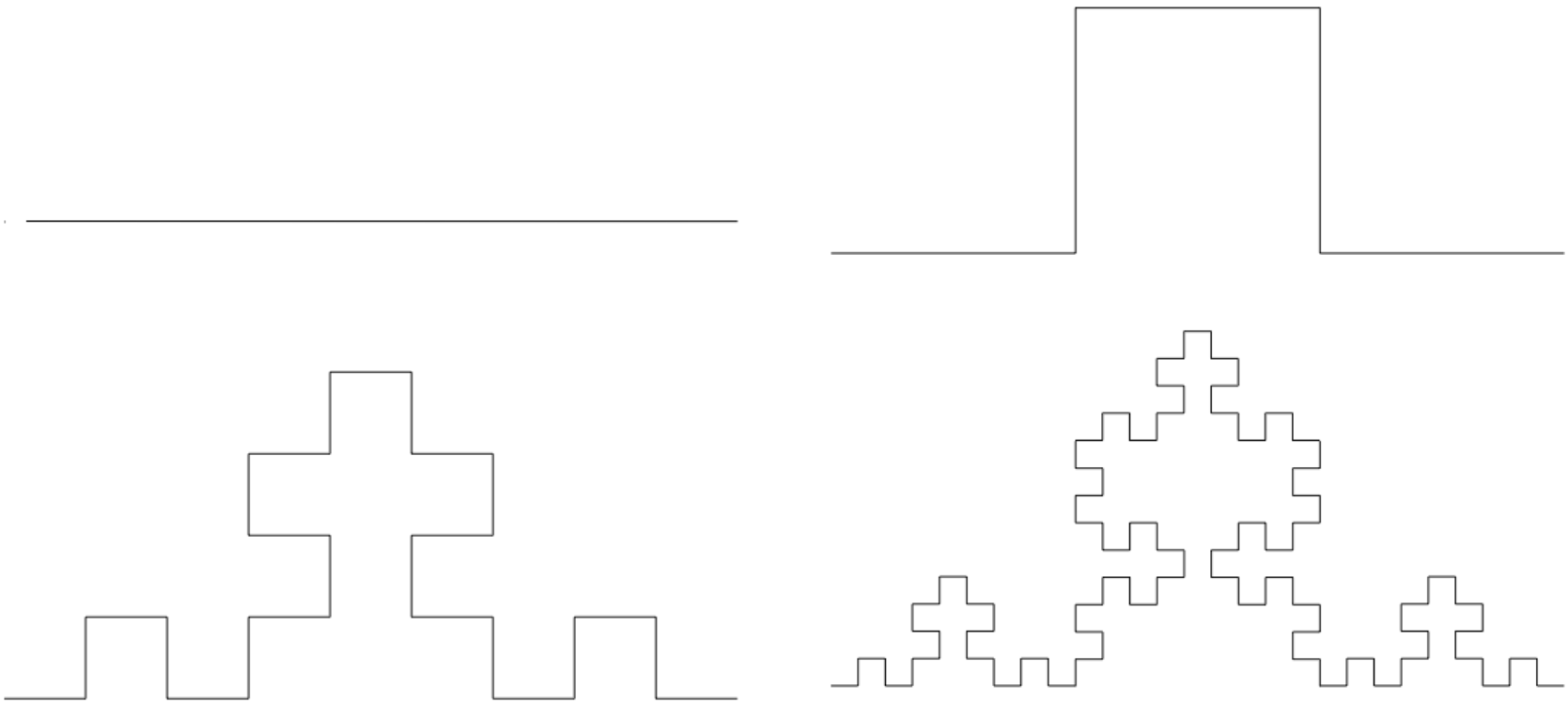


Building a self-similar fractal



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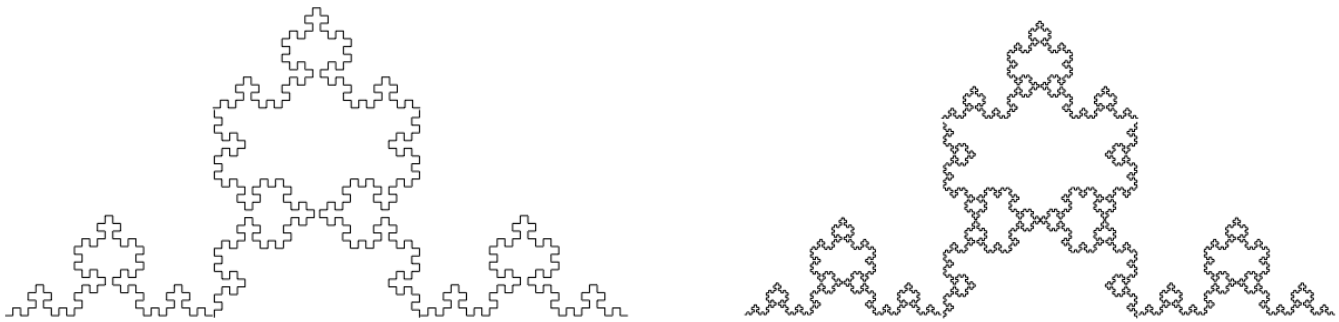

- A simple rule creates a complicated pattern.

The Koch curve

- This fractal is so well known it has a name.
- Try it!

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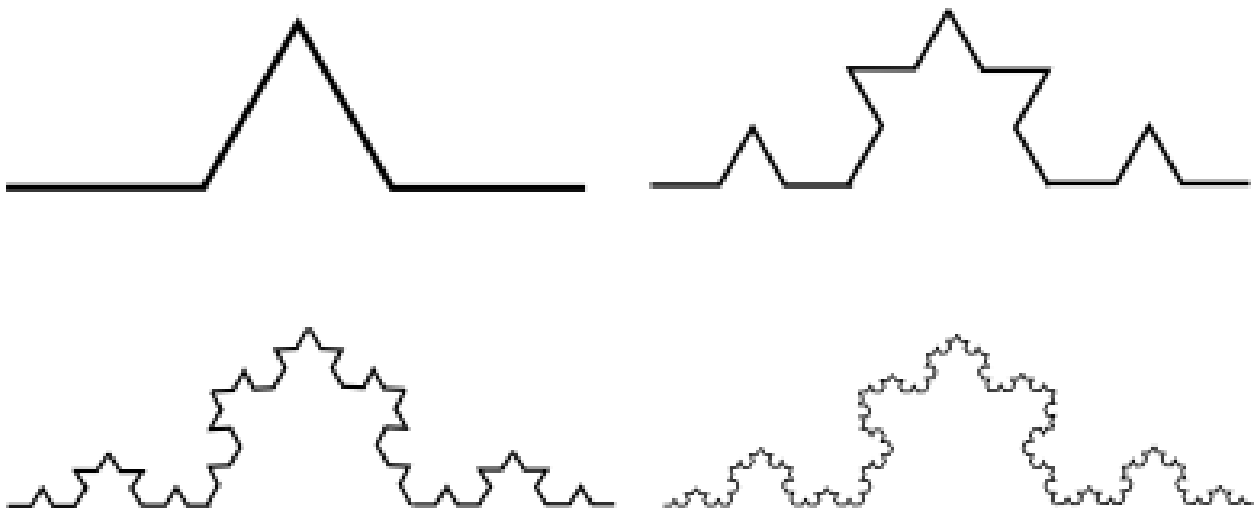


Inifinitely many times?

- This could take a while....
- Luckily, since the component lines become smaller and smaller, they are soon below screen/eye resolution.
- At this stage, the 'next step' looks almost exactly the same.

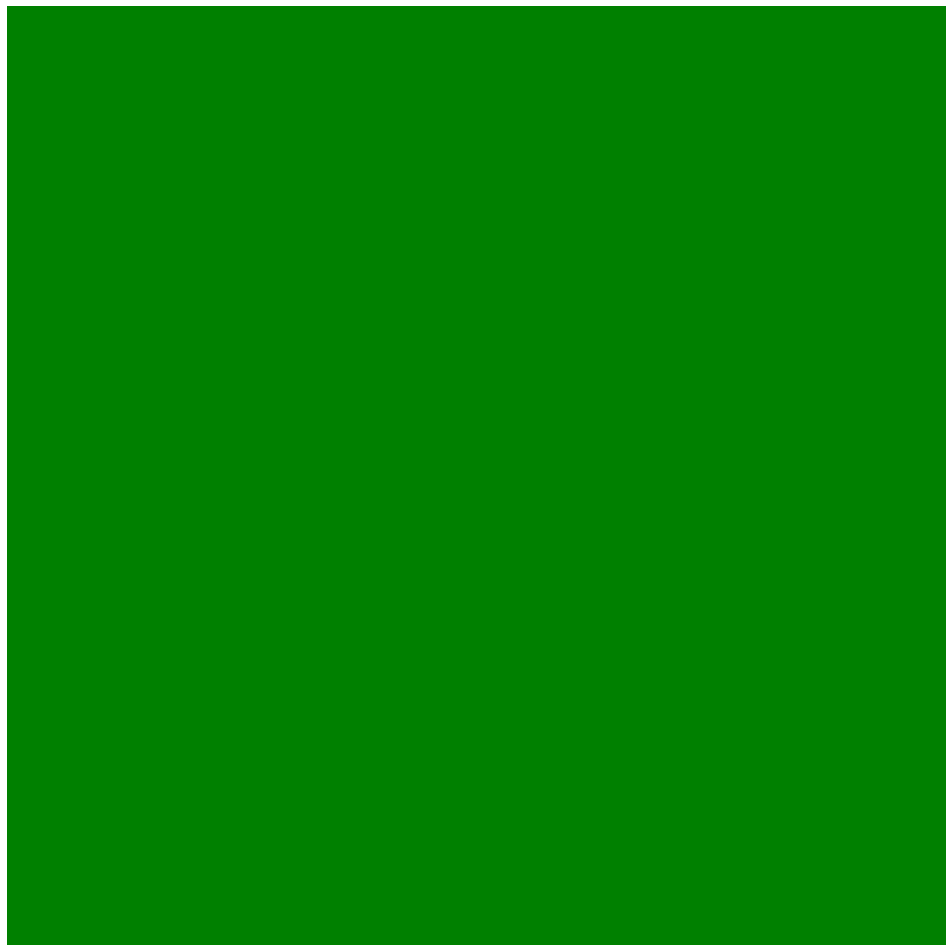
How long?

- In the first step, a line of length 1 is replaced by four lines of length 1/3.
- At each step the length of the line increased by a factor of 4/3.
- So at the second step, the line has length 16/9.
- So...

- The Koch curve has infinite length!

Fractals from areas

We can generalise this method of building fractals: instead of replacing each line segment with a group of smaller line segments, we can replace other geometric objects with smaller objects.



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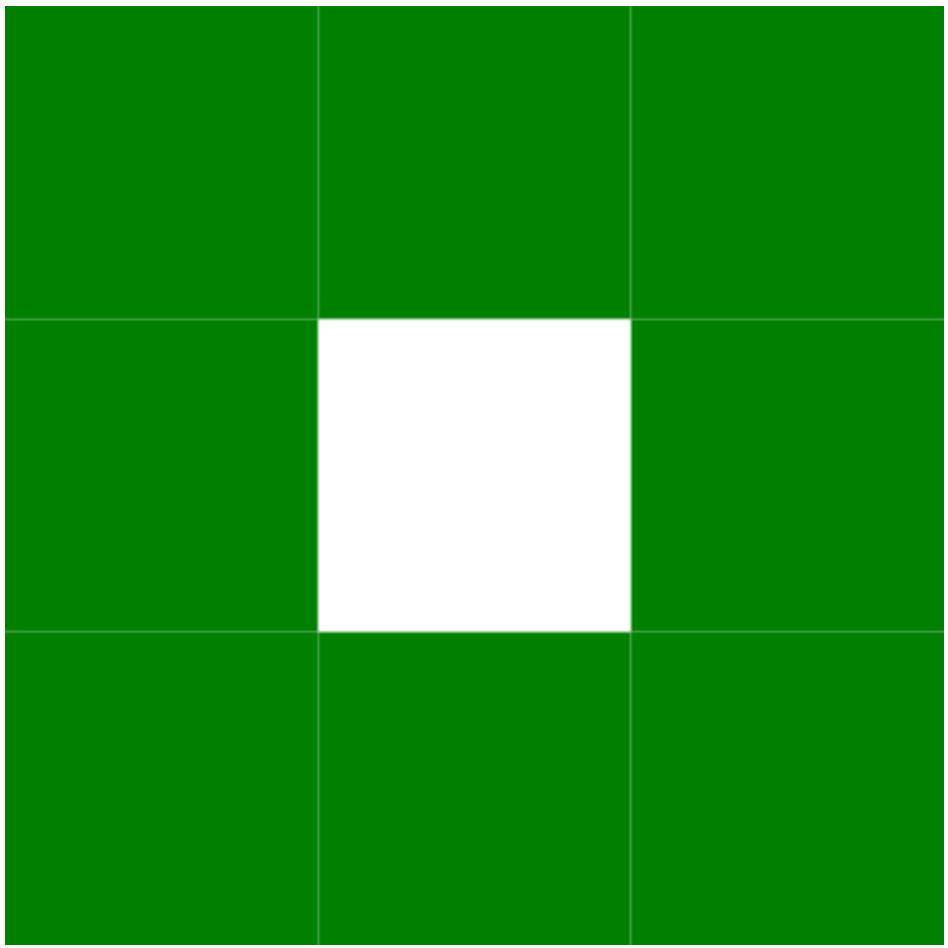

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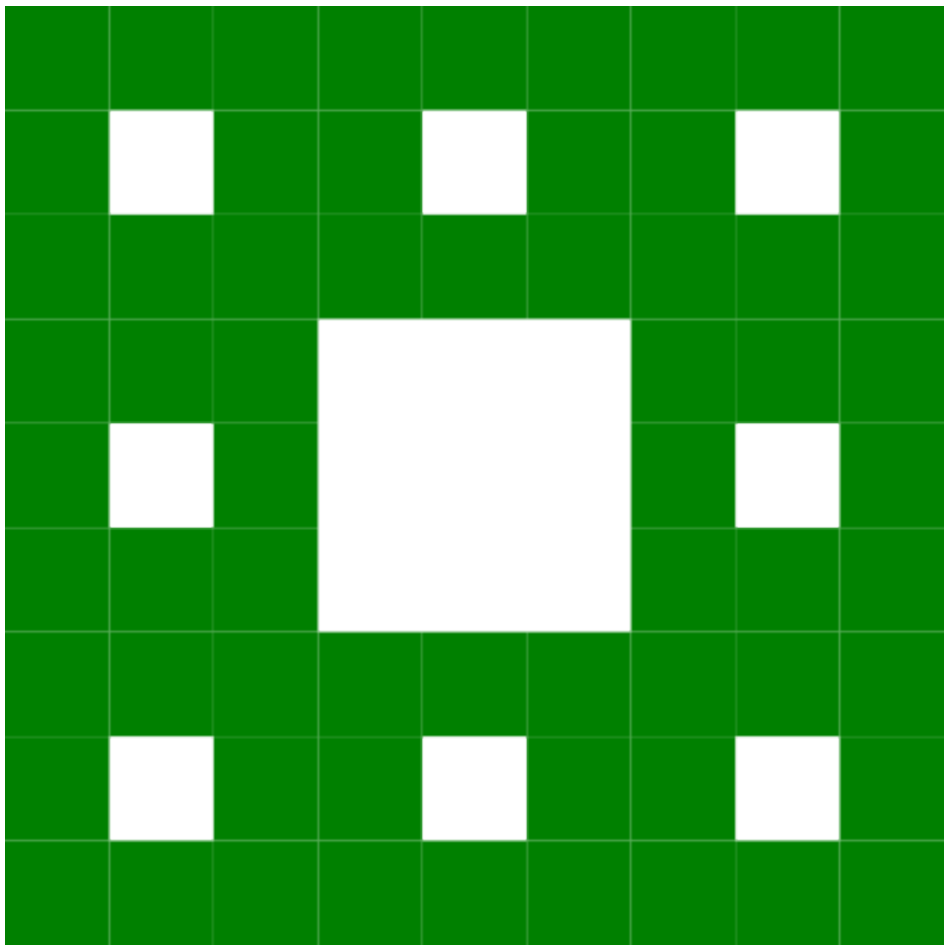

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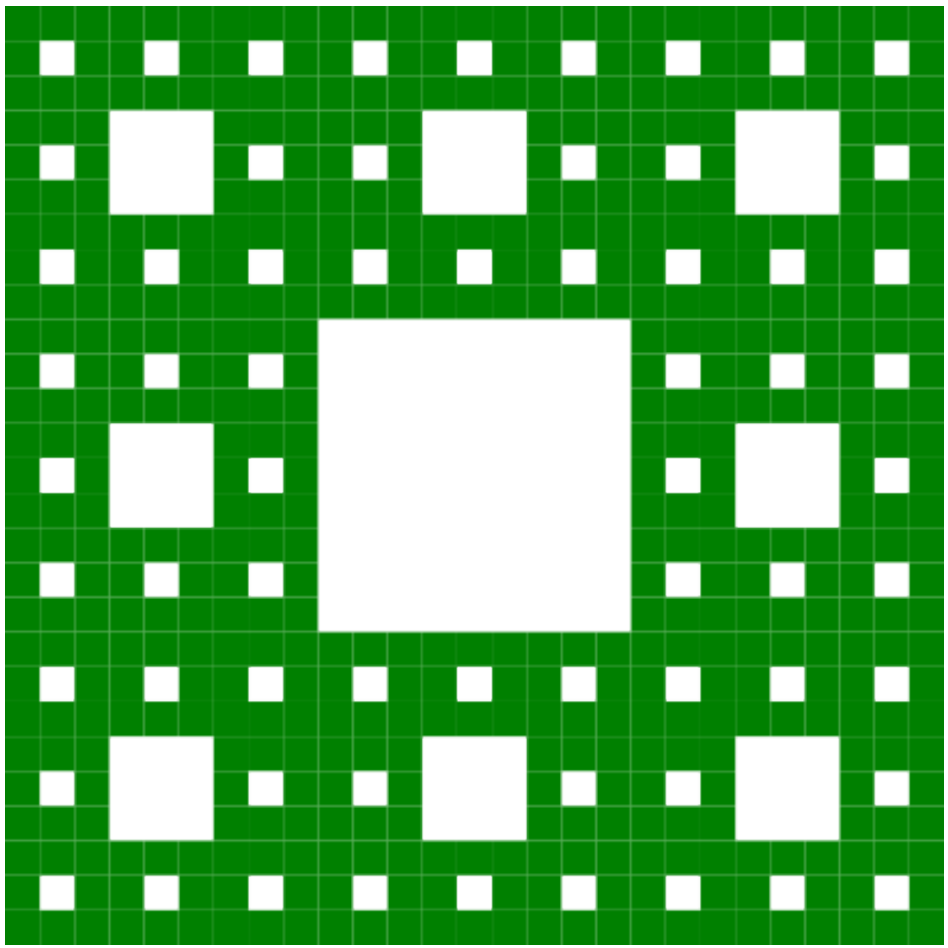

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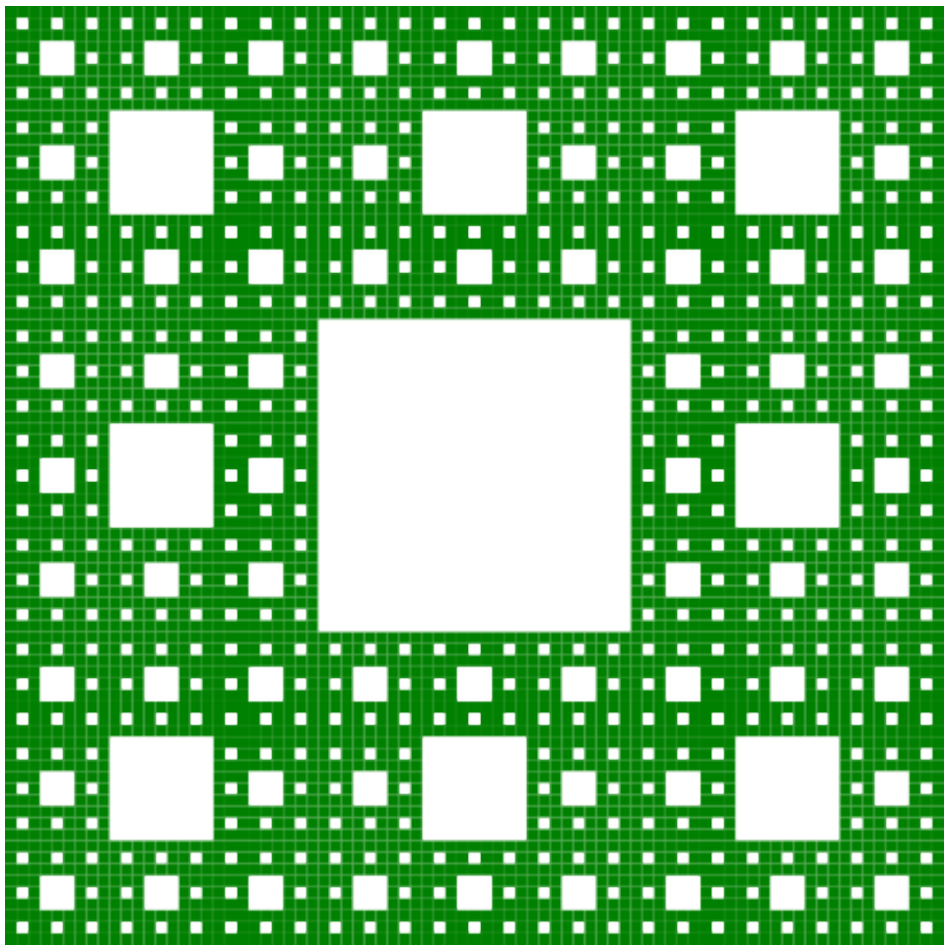

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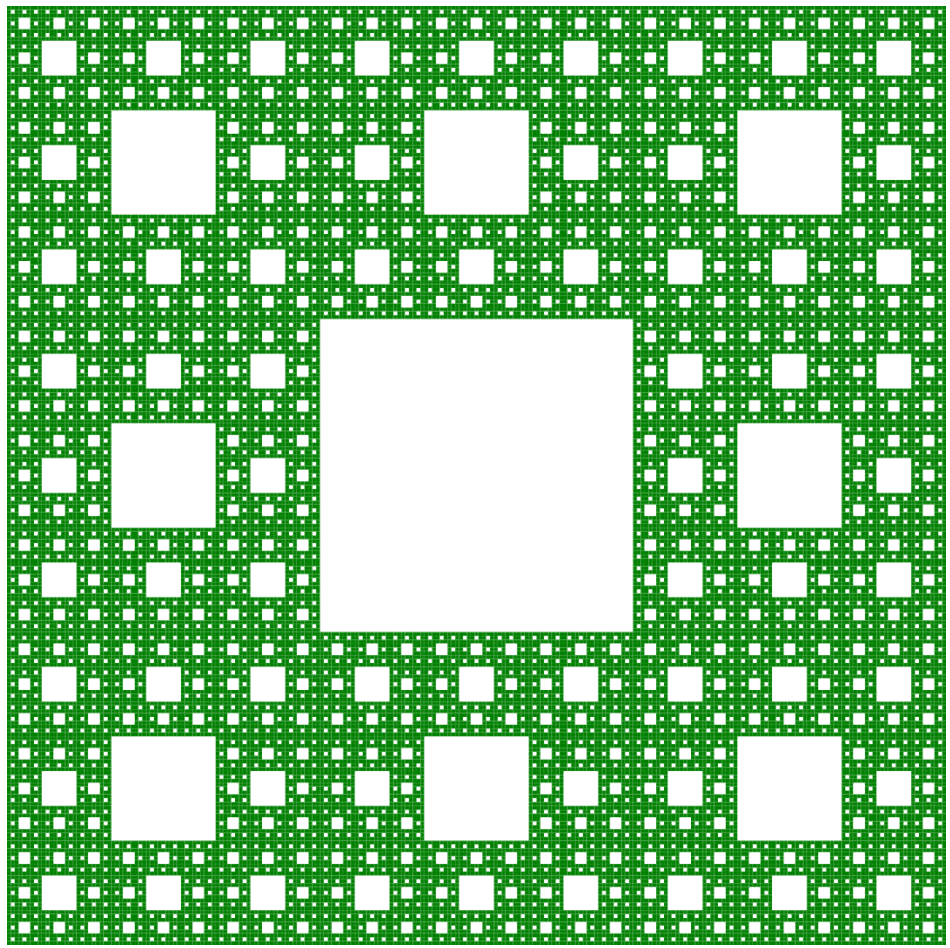
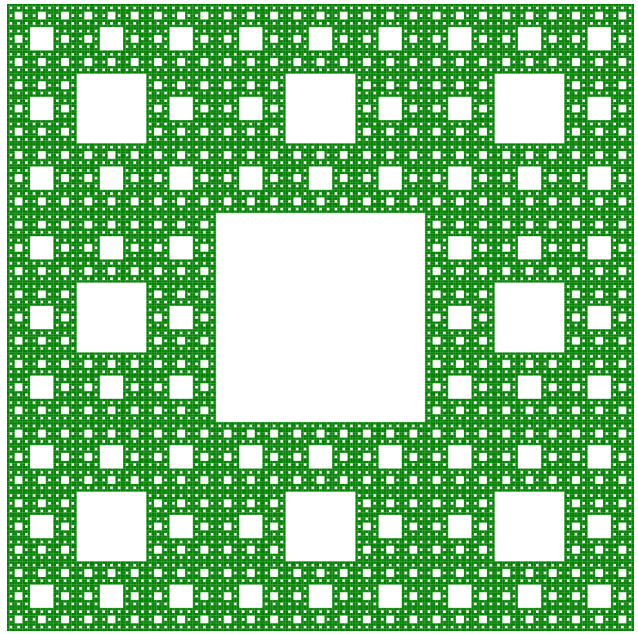


Seirpinski Carpet



- What is the area?
- At each stage, each square is replaced by 8 smaller squares, each 1/9 of the area of the original square.
- Each step has area 8/9 of the previous step.

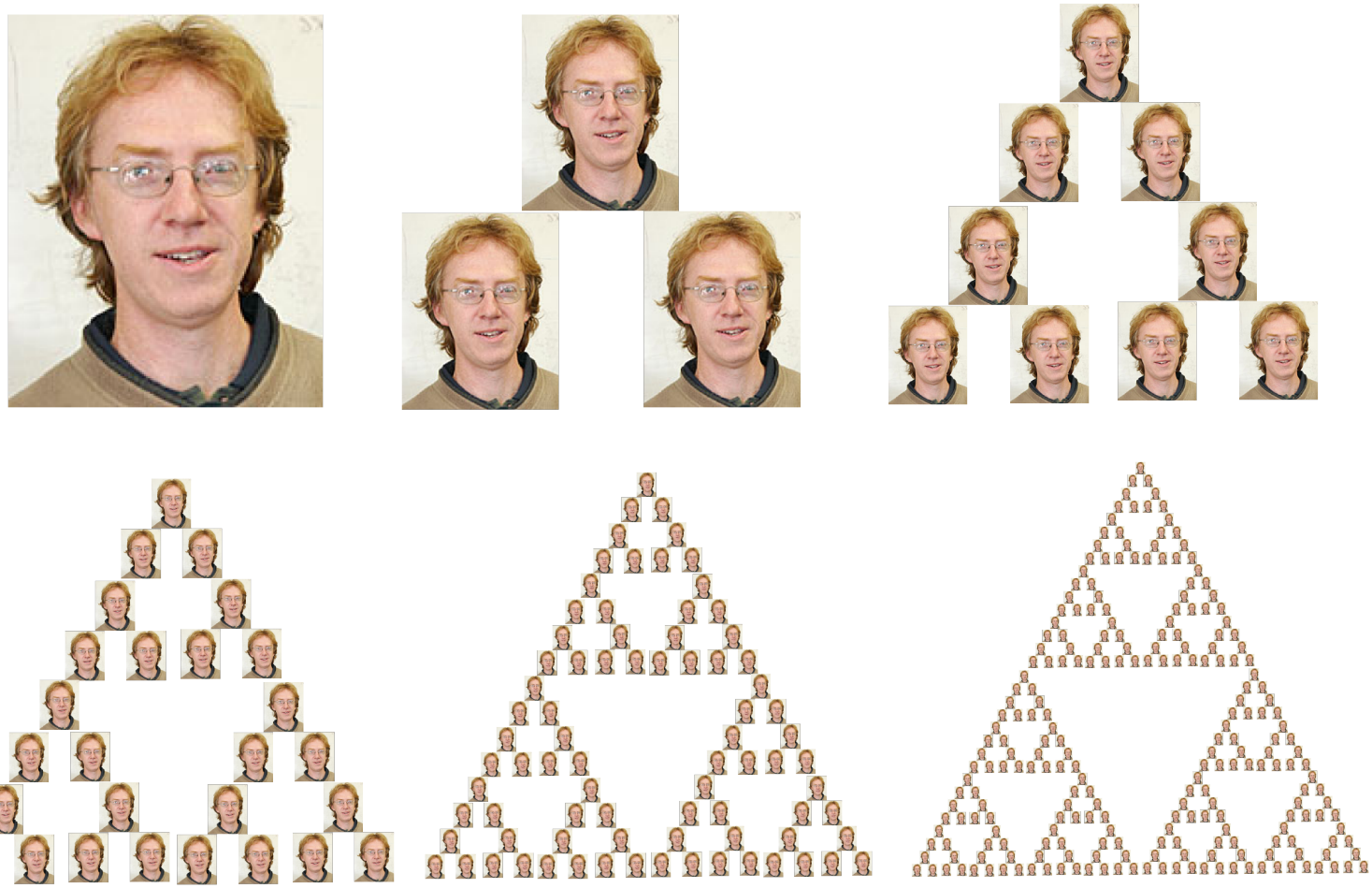
- As we head towards infinity, the area remaining tends to zero!

Building fractals by collage

Procedure:

1. Start with a picture.
2. Make some number of copies, each reduced by a specified amount.
3. Position each reduced picture on a page in a specified position, creating a new picture on the page (it is a collage of pictures).
4. Using the image you got in Step 3, go back to Step 1 and repeat the process, repeating the cycle forever.

A fractal built by collage



Another fractal built by collage: Barnsley's fern

Important ideas from today:

- Simple repeated processes can lead to complex and interesting outcomes.
- Fractals appear in nature, and can be constructed by computer or even by hand.
- Fractals can have infinite length, or zero area.