Maths 190 Lecture 2

- **Topic for today: Counting!**

- **Vitally important question**:

  Do there exist two nonbald people on the planet who have exactly the same number of hairs on their body?

Tennis Balls

- How many tennis balls would it take to fill this room?

- How would you estimate how many grains of sand are on the beach?

Tennis Balls again

- I have six tennis ball cans. Each can can hold at most four tennis balls.
- How do I know that at least two cans contain the same number of tennis balls?

Back to our important question...

- How may hairs do you have on your body?
- Make a rough estimate on a small area.
- Find an upper limit.

Finding an upper limit

- Roughly 100 hairs in 5mm $\times$ 5mm square.
- So about 400 hairs in a square centimetre.
- *Surely* no-one can have more than 4000 per square centimetre.

- Estimate surface area as a cylinder: 2m high, 100cm around: area of $200 \times 100 = 20,000$ square centimetres.
- *Surely* no-one can have a surface area greater than 200,000 square centimetres.

Maximum number of hairs

- So a person 10 times as hairy (all over!), with 10 times the surface area, would have at most

$$200,000 \times 4000 = 800,000,000 \text{ hairs}$$

How many people?

- Current estimates put the world population at about 6.6 billion

$$6,600,000,000 \text{ people}$$

What do these two numbers tell us?

- At most 800,000,000 hairs on a person.
- At least 6,600,000,000 people in the world.

- Two must have exactly the same number of hairs....
- But we can't tell who these people are.

The Pigeonhole Principle

- If we have a collection of things to put in categories, and there are more things than categories, then one category must contain more than one thing.

More mathematically

- If there are k times more things than categories, then one category must contain *at least k* or more things.
- So actually, since

$$\frac{\text{people}}{\text{hairs}} = \frac{6,600,000,000}{800,000,000} = 8.25$$

  there must be at least *nine* people in the world with the same number of hairs.
- Note that we don't know which category has k things - that is, we don't know how many hairs the nine people have.

Some notes on language

Suppose we divide 13 sweets between 3 people, Claire, Steven and James.

- Claire: 4, Steven: 2, James: 7

What is the largest number you can write in the box?

- One of Claire, Steven and James has at least $\square$ sweets.
- Each of Claire, Steven and James have at least $\square$ sweets.

Some notes on language

Suppose we divide 13 sweets between 3 people, Claire, Steven and James.

- Claire: 13, Steven: 0, James: 0

What is the largest number you can write in the box?

- One of Claire, Steven and James has at least $\square$ sweets.
- Each of Claire, Steven and James have at least $\square$ sweets.

Some notes on language

Suppose we divide 13 sweets between 3 people, Claire, Steven and James.

- Claire: ?, Steven: ?, James: ?

What is the largest number you can write in the box, and be sure of being correct, however the sweets are divided?

- One of Claire, Steven and James has at least $\square$ sweets.
- Each of Claire, Steven and James have at least $\square$ sweets.

A harder example

The numbers $1, 2, \ldots, 8$ are written in a circle, in any order. Show that there are 3 adjacent numbers whose sum is 14 or greater.

Important ideas from today:

- Its possible to estimate very large numbers of things by breaking one large problem down into a lot of smaller ones.
- The pigeonhole principle. This principle says that if you have to put N things into N-1 boxes, at least one box has to have two things in it.

For next time

- Read 2.2 in the text and think about other patterns you see in nature, or around you in your daily life.
- Bring **a pineapple** to the next class.