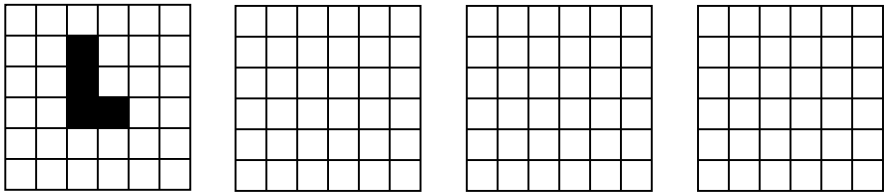
DEPARTMENT OF MATHEMATICS

MATHS 190 Tutorial 9

Tutorials in Maths 190 are **collaborative tutorials**. You should work in groups of 3 or 4 students, discussing the situations and puzzles listed below, or issues arising from lectures. Part of your final mark depends on your participation in tutorials.

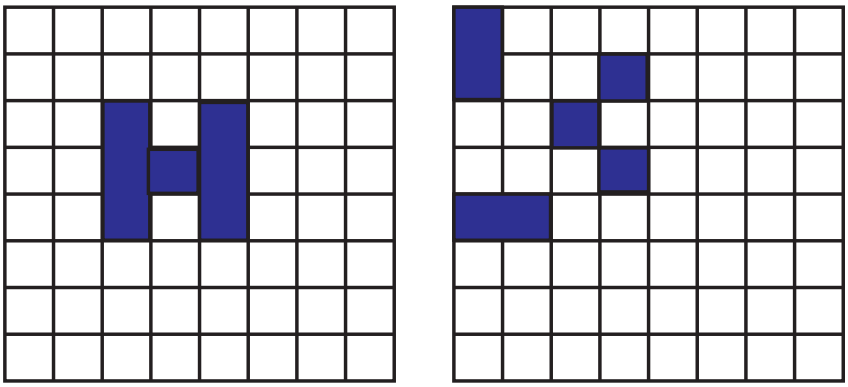
1. Suppose that a population is modelled using the model, $P_{n+1} = cP_n(1 - P_n)$. Suppose that the initial population density P_1 is 0.25 and the next year's population density P_2 is 0.5. Find the value of the constant c. Use your answer to compute P_3 and P_4.

2. For the following initial population in the Game of Life, find the next three generations.



3. **Write up your answer to this question and hand it in with your answers to Assignment 4 (due October 21st).** See below for instructions on writing tutorial reports. Don't forget to write down the names of the people in your tutorial group, so that you can acknowledge your collaborators in your report.

   For each of the initial populations in the Game of Life find out the behaviour of the population — compute the next few generations and decide if the population will explode, go extinct, become periodic etc. Use the templates handed out in the tutorial. You can also get copies of these from the course website.



4. Here is an iterative dynamical system: given x_n, then

$$x_{n+1} = f(x_n) = \begin{cases} 2x_n & \text{if } x_n < 1/2 \\ 2 - 2x_n & \text{if } x_n \geq 1/2 \end{cases}$$

   (a) If $x_0 = 2/7$, compute x_1, x_2 and x_3. What do you observe? What will be the long term behaviour if you keep on iterating?

   (b) Calculate the first four iterations if $x_0 = 0.01$. What do you observe? What will be the long term behaviour if you keep on iterating?

   (c) Calculate the first four iterations if $x_0 = 2/3$. What will be the long term behaviour if you keep on iterating?

   (d) Calculate the first four iterations if $x_0 = 2/3 + 0.01$. What do you think will happen to other iterations started at x_0 near $2/3$?

5. **Harder!** A French chef made a masterpiece with a segment of dough one unit long. He noticed that there were three grains of sugar embedded in the dough; one red, one white, one blue. He stretched the dough to twice its length and then folded it in half to produce a double layer of thinner pastry still one unit long. He did this again and again. He noticed that the red grain always returned to its original position after each stretch and fold. The white grain returned to its original position after two stretches and folds, while the blue grain took three stretches and folds to return to its original position (see the picture). Where did the red, white and blue grains start?

Writing up tutorial questions

A written solution for question 3 should be handed in for marking with Assignment 4. In your solution, you should include:

- the names of the people you discussed this with in your tutorial group;
- a clear statement of your solution to the puzzle;
- a clear explanation (in one or two paragraphs) or how you arrived at this solution; and,
- a statement of any assumptions you had to make in obtaining your answer.