

- ▶ Topic for today: **Cyclical arithmetic**
- ▶ **Vitally important question of the day:**

What do clocks and bar codes have in common?

What time will it be in 50 hours time?



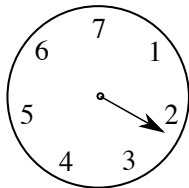
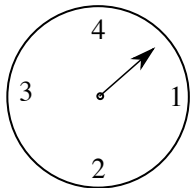
Clock arithmetic



- ▶ $2 + 4 =$
 - ▶ $10 + 4 =$
 - ▶ $7 + 7 =$
 - ▶ $3 - 5 =$
-
- ▶ Suppose it takes you 13 hours to do one problem from your Maths 190 assignment. If you start at 1 o'clock and there are 5 problems, what time will it be when you are finished (assuming you work straight through)?

Crazy clocks

- ▶ There is no good reason why our clock is divided into 12 hours.
- ▶ Do the following sums on the 4 hour clock and the 7 hour clock:



- ▶ $2 + 4 =$
- ▶ $3 - 4 =$
- ▶ $7 + 7 =$
- ▶ $3 - 5 =$
- ▶ $1 + 13 \times 5 =$

Modular arithmetic

- ▶ ' $m \bmod n$ ' means 'the remainder when I divide m by n '.
- ▶ We do modular arithmetic when we calculate the time.
- ▶ We define equivalences

$$a \equiv b \pmod{n}$$

Calculations mod 10

- ▶ Doing calculations means using equivalences:

$$10 \equiv 0 \pmod{10}$$

- ▶ We can simplify expressions as we go along:

$$\begin{aligned} & 12 + 34 + 67 \pmod{10} \\ &= 2 + 4 + 7 \pmod{10} \\ &= 13 \pmod{10} \\ &= 3 \pmod{10} \end{aligned}$$

- ▶ The same thing works for multiplication

$$\begin{aligned} & 37 \times 16 \pmod{11} \\ & = (3 \times 11 + 4) \times (11 + 5) \pmod{11} \\ & = 4 \times 5 \pmod{11} \\ & = 20 \pmod{11} \\ & = 9 \pmod{11} \end{aligned}$$

- ▶ And for taking powers:

$$\begin{aligned} & 33^5 \pmod{8} \\ & = (4 \times 8 + 1)^5 \pmod{8} \\ & = 1^5 \pmod{8} \\ & = 1 \pmod{8} \end{aligned}$$

Check digits

- ▶ Find something with a bar code
- ▶ There should be 13 digits; if there are only 12, imagine there's a zero in front.
- ▶ This book: 9 781931 914413
- ▶ But not every 13 digit number is a bar code.

A check sum

- ▶ Call the digits d_1 to d_{13}
- ▶ Multiply the even-numbered digits by 3, and then add them all together:

$$d_1 + 3d_2 + d_3 + 3d_4 + d_5 + 3d_6 + d_7 + 3d_8 + d_9 + 3d_{10} + d_{11} + 3d_{12} + d_{13}$$

- ▶ For my example, I get:

$$9 + (3 \times 7) + 8 + (3 \times 1) + 9 + (3 \times 3) + 1 + (3 \times 9) + 1 + (3 \times 4) + 4 + (3 \times 1) + 3$$

- ▶ Then compute the result mod 10.
- ▶ How can you tell if there's a mistake?

Try it!

- ▶ Get into pairs.
- ▶ Taller student: find a 13 digit bar code.
- ▶ If your birthday is on an even date, write down the bar code *with one digit incorrect*.
- ▶ If your birthday is on an odd date, write down the bar code *correctly*.
- ▶ Shorter student: is your partner's birthday on an odd or even date?

ISBN numbers

- ▶ 11-digit ISBN numbers use $\pmod{11}$ (using $X = 10$)
- ▶ $d_1 + 2d_2 + 3d_3 + 4d_4 + 5d_5 + 6d_6 + 7d_7 + 8d_8 + 9d_9 + 10d_{10} = 0 \pmod{11}$
- ▶ This book: ISBN 1-931914-41-9

Fermat's little theorem

- ▶ If p is a prime number and n is any integer which does not have p as a factor, then

$$n^{p-1} = 1 \pmod{p}$$

- ▶ Try it!
- ▶ $p = 5$, try $n = 3$, $n = 7$, $n = 9$.

Important ideas from today:

- ▶ When we compute the time, we do modular arithmetic.
- ▶ Modular arithmetic is also used in bar codes and ISBN numbers (among other things).

For next time

- ▶ Read Section 2.6 of the textbook