Regular maps and triangle groups

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Abstract

Regular maps on surfaces are geometric patterns admitting algebraic descriptions. More precisely, a map is a 2-cell decomposition of a closed compact two dimensional manifold. A map is regular if its automorphism group acts transitively on the vertex-edge-face tuples of the map.

Their classification is at a crossroads in pure mathematics, drawing from the theory of Riemann surfaces, combinatorial group theory and topological graph theory. Two important invariants used for classification are: the genus and orientability of the supporting surface, or their automorphism group. Much contemporary work has been done on pursuing the classification by genus.

The geometric picture is misleading since regular maps live fruitful lives as purely algebraic objects. They correspond to torsion-free normal subgroups of Dyck's triangle groups. Work by Conder and Dobscanyi exploits the algebraic structure of the triangle groups, and the 'Low Index Subgroups' algorithm in MAGMA, to exhaustively search for such subgroups.

Using this data, we have been able to identify several simple families of regular maps. A family of regular maps corresponds to a (non-degenerate) parameterized presentation of the torsion-free subgroups. My project uses classical techniques from combinatorial group theory to give constructions for these families. I will give an overview of milestones in the classification, some examples I have identified as part of my research and other directions in related research.