

Maths 190 Assignment 2

August 16, 2009Due: 4pm, August 26 2009

- Put your completed assignment in the appropriate box in the basement of the Mathematics/Physics Building **before** 4pm on the date due.
 - Late assignments or assignments placed in the wrong box will not be marked.
 - Your assignment **must** be accompanied by a blue Mathematics Department coversheet. Copies of the coversheet are available in the basement.
1. (5 marks) Prove that $\sqrt{2} + \sqrt{3}$ is irrational.
 2. (5 marks) Zaphod lives on the planet Betelgeuse, and he is struggling with his Maths homework. On Betelgeuse, the days are 9 hours long, there are 7 days in the week (with the same names as our days, strangely) and 353 days in each year.
 - (a) Zaphod has 6 maths problems to do. Each problem takes him 4 hours to complete. If he starts working at 3 o'clock, assuming he works straight through, what time will it be when he finishes?
 - (b) This year Zaphod's birthday was on a Tuesday. What day will it be on next year?
 - (c) Zaphod's cousin Ford starts cutting up a piece of paper. First he cuts it into 4 pieces. Then he selects one of the four pieces and cuts that into 4 pieces, so there are 7 pieces of paper in total. Then he selects one of the 7 pieces and cuts it into 4 pieces. He continues doing so (selecting a piece of paper at random from the whole pile and cutting it into 4 pieces) whilst Zaphod is finishing his homework. At the end, Ford counts the total number of pieces of paper and announces that there are 3330 pieces of paper. How do you know that Ford made a mistake somewhere along the line?
 3. (3 marks) Carefully show that the set of even integers $E = \{\dots - 4, -2, 0, 2, 4, \dots\}$ has the same cardinality as the set of natural numbers $N = \{1, 2, 3, 4, 5, \dots\}$.
 4. (5 marks) Suppose Vivien has infinitely many packages of chocolate fish and chocolate frogs in her office, one package of fish for each positive integer and also one package of frogs for each positive integer. Each package of chocolate frogs contains 14 frogs and each package of chocolate fish contains 6 fish. Does Vivien have more frogs than fish in her office? Explain your answer carefully.
 5. (7 marks) Let D be the set consisting of all numbers between 1 and 2 which have decimal expansions that use only the digits 1 and 2. So, for example, the number 1.1212121212.... could be in the set D but the number 1.123123123..... is not. Carefully show that the cardinality of D is greater than the cardinality of the set of natural numbers.

Tutorial write up: Remember to hand in with your assignment your written solutions to question 5 on tutorial 3 (5 marks), question 7 on tutorial 4 (5 marks), and question 5 on tutorial 5 (5 marks).