DEPARTMENT OF MATHEMATICS

MATHS 260 **Tutorial 7**

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1. (a) Write the following system of DEs in vector form:

$$\begin{aligned} \frac{dx}{dt} &= 2x + 2y \\ \frac{dy}{dt} &= x + 3y \end{aligned}$$

(b) The eigenvalues of the coefficient matrix are 4 and 1 with the associated eigenvectors $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ respectively. Write down formulae for the straightline solutions of the DE.

(c) For each straight-line solution use `analyzer` to plot x as a function of t and y as a function of t, both on the same picture.

(d) Using the straight-line solutions, sketch a phase portrait for the system. Use your notes from Lecture 18 to help you.

(e) Write down the general solution, and use your answer to find the solution to the IVP consisting of the DE and the initial condition

$$\mathbf{Y}(0) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}.$$

(f) Using `pplane`, draw the direction field for the system and some representative solution curves, including the straight-line solutions. Compare this to the phase portrait you sketched in (d).

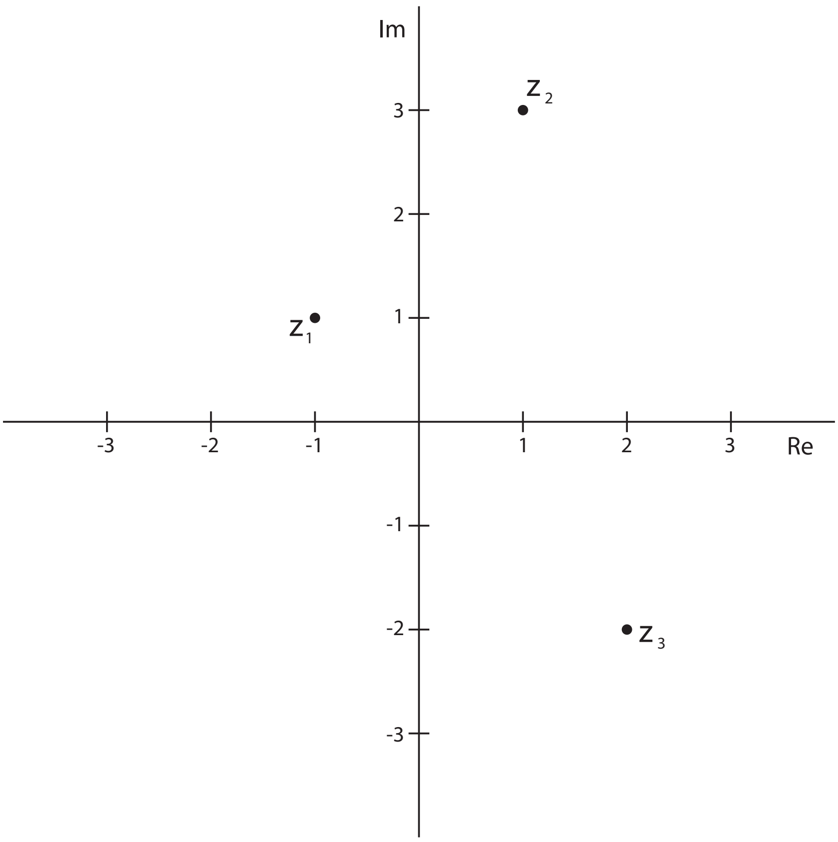
(g) Use `pplane` to plot the functions $x(t)$, $y(t)$ for each straight-line solution. This is done under the `Graph` menu - choose `Both` - then click the cursor on the straight-line solution. Compare the graphs with those you obtained in part (c) above.

2. (a) Change $z_1 = -2 + i$ and $z_2 = 1 + 3i$ into polar form.

(b) Change $z_1 = 2e^{i\pi/3}$ and $z_2 = \sqrt{2}e^{i5\pi/4}$ into rectangular form.

(more questions over page)

3. For the Argand Diagram below;



(a) Write the complex numbers z_1, z_2 and z_3 in rectangular and polar form.

(b) Using the rectangular form, calculate $z_1 z_2$ and z_3/z_1. Add these points to the Argand Diagram.

(c) Using the polar form, calculate $z_1 z_2$ and z_3/z_1. Check these results match the points on the Argand Diagram from part (b).

4. **Challenge question:**

(a) Consider a linear differential equation

$$\frac{d\mathbf{Y}}{dt} = \mathbf{AY}.$$

Show that if $\mathbf{Y}(t) = \mathbf{Y_1}(t) + i\mathbf{Y_2}(t)$ is a solution then $\mathbf{Y}(t) = \mathbf{Y_1}(t)$ and $\mathbf{Y}(t) = \mathbf{Y_2}(t)$ are also solutions.

(b) Consider the system of differential equations

$$\dot{x} = -x + 2y \tag{1}$$

$$\dot{y} = -2x - y \tag{2}$$

Show that $x(t) = e^{-t}\cos(2t) + ie^{-t}\sin(2t)$, $y(t) = -e^{-t}\sin(2t) + ie^{-t}\cos(2t)$ is a solution. Hence find two linearly independent *real* solutions to the equations.