DEPARTMENT OF MATHEMATICS

MATHS 260 **Tutorial 11**

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1. Two Martian species, the Yoiks and the Zarks, live in close proximity on Mars. They share the same basic requirements so they are in constant competition for resources. Such a situation can be modelled with the following system of equations:

$$\frac{dY}{dt} = sY(1 - aY - bZ)$$
$$\frac{dZ}{dt} = rZ(1 - cY - dZ)$$

   where $r, s, a, b, c, d > 0$. Here, $Z(t)$ is the population of Zarks at time t and $Y(t)$ is the population of Yoiks at time t. For the model to be realistic, we must have $Z(t) \geq 0$ and $Y(t) \geq 0$ for all t.

   (a) Briefly describe the physical significance of each term in the model.

   (b) For the choice of parameters $r = s = 1$, $a = 3$, $b = 1$, $c = 1$, $d = 2$, sketch the phase portrait by following the steps:

      i. Find all the equilibrium solutions and determine their type. What do each of these states represent physically?

      ii. For each equilibrium solution, sketch a phase portrait showing the behaviour of solutions near to that equilibrium.

      iii. Find and sketch the nullclines for this system.

      iv. Carefully draw a phase portrait for this system (by hand).

   (c) What happens to the populations of Zarks and Yoiks in the long term? Your answer should cover all possible initial conditions.

2. Write each of the following higher-order differential equations as a system of first-order differential equations.

   (a) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + y = 0$

   (b) $\frac{d^2x}{dt^2} + x\left(\frac{dx}{dt}\right)^2 = 2$

   (c) $\frac{d^3y}{dt^3} = \frac{dy}{dt}$

3. Find the general solutions of the following equations.

   (a) $\frac{d^2y}{dt^2} + \frac{dy}{dt} - 12y = 0$

   (b) $\frac{d^3y}{dt^3} - 4\frac{dy}{dt} = 0$

4. **Challenge Question:** Consider the following second-order differential equation

$$\frac{d^2x}{dt^2} = kx + \frac{dx}{dt} - x^3 - x^2\frac{dx}{dt}$$

where k is a parameter.

(a) Write the equation as a system of first-order differential equations.

(b) Use any methods you like to investigate the behaviour of the system as the parameter k changes. *Hint: you should find five bifurcations for* $k \in [-2, 2]$. *You will have to look closely though!*