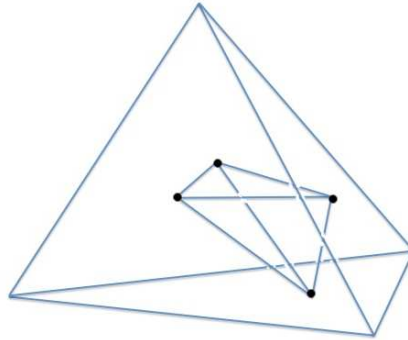


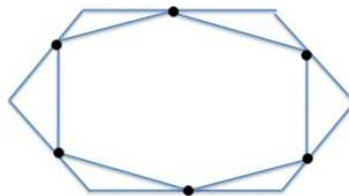
# Maths 190 Assignment 3 Solutions

April 30, 2010

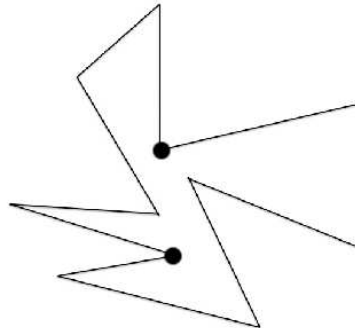
1. (a) (2 marks) Add vertices in the center of each of the 4 faces. There are 4 vertices. Since each face in the original solid touches all others, we draw edges in the dual between all pairs of vertices. We therefore draw a tetrahedron.



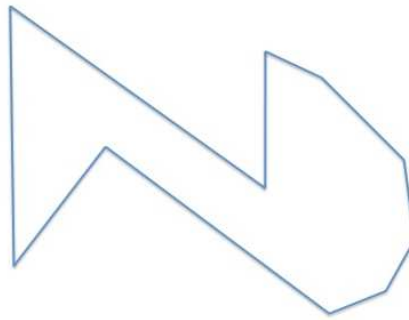
- (b) (2 marks) There are  $n$  edges in the original polygon so the dual has  $n$  vertices. Each edge is connected to two vertices, so there are 2 edges out of each vertex in the dual, and these edges form a closed polygon. Hence, the dual of a regular polygon on  $n$  vertices is a regular polygon on  $n$  vertices.



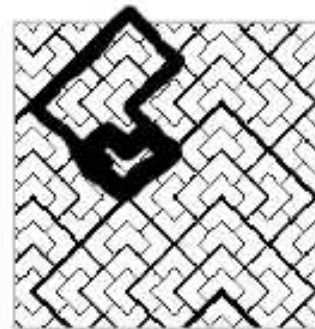
2. (a) (2 marks) Two guards are needed.



- (b) The following picture is the floor plan of a gallery with  $n$  (in this case  $n = 10$ ) which needs exactly 2 guards. The “bulge” on the right hand side can be any convex polygon with  $n - 6$  vertices.



3. (a) (1 mark) A supertile is a tile of exactly the same shape as the original tile, which is formed out of copies of the original tile.
- (b) (1 mark) A pattern in the plane has a **symmetry of scale** if the tiles that make up the pattern can be grouped into super-tiles that still cover the plane and, if scaled down, can be rigidly moved to coincide with the original pattern.
- (c) (2 marks) The supertiles are crudely marked in the following picture.



4. (a) (2 marks)



- (b) (1 mark) The picture is made of 3 pieces, all of which are  $1/3$  the size of the original. Hence, the fractal dimension is

$$\log(3)/\log(3) = 1.$$

### Tutorial questions:

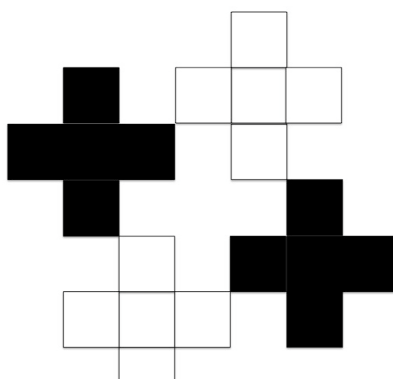
Note: the marks for these need to be recorded separately from the assignment marks.

**Tutorial 6, Question 4:** (5 marks, 1 for each correct case)

The boundary of the cut in each case is: triangle (for the tetrahedron), triangle (for the cube), square (for the octahedron), triangle (dodecahedron), pentagon (icosahedron). In all cases this is the polygon whose number of edges is equal to the number of faces of the original solid which meet at a vertex.

**Tutorial 7, Question 5:** (5 marks)

One way to describe the tiling is as infinite diagonal rows: the next tile sits two squares to the right and one down from the previous tile. Once one row is made one can continue the tiling by placing a tile above (or below) the row and performing the same process again. The following image gives a small portion of such a tiling.



The tiling is preserved by **shifts** right or left by 5 squares, up or down by 5 squares, right two squares and down one (and its reverse) and up two squares and right one (and its reverse).

The tiling is not preserved by any **flips** (reflections). This can be seen by observing that a flip can only be of two types: a flip along a line which forms one of the edges of a plus, or a flip along a vertical, horizontal or diagonal line which cuts the plus into equal halves. In each case one can see that the flip does not map the tiling to an exact copy of itself.

The tiling is preserved by **rotations** of 90, 180 or 270 degrees around the center of any plus or around a vertex where four “outer” corners meet (i.e., the points in the tiling which form a “swastika” shape).

[Please note that the swastika is an ancient symbol which is used in the Hindu and Buddhist religions, and should not be tainted by its use by the Nazi’s.]

The tiling does not have any **symmetries of scale**. If one tries to make a big plus sign out of little ones then the outer corners will always have a “bite” taken out of them.

**Tutorial 8:** (5 marks) There are lots of possible answers. The important point is that in going from the zeroth to first step (or first to second step), 7 copies of the original shape should be used, each scaled down by a factor of 4.

The easiest way to draw it is to start with a  $4 \times 4$  square and colour any 7 of the original 16 squares.