Maths 260 Lecture 1

- **Topics for today:** Introduction to differential equations
- **Reading for this lecture:** BDH Section 1.1
- **Suggested exercises:** BDH Section 1.1: 1, 3, 13, 15
- **Reading for next lecture:** BDH Section 1.2
- **Today's handouts:** Course studyguide

Modelling and Differential Equations

- The subject of differential equations is about using derivatives to describe how a quantity changes.

- Using knowledge about how a quantity changes to write down a DE is called **modelling**, and a DE is a **model.**

- The goal of modelling is to use the DE model to predict future values of the quantity being modelled.

Example 1: Single Population, Unlimited Growth

Assume: Population grows at a rate proportional to the size of the population

Quantities of interest:
t=time
P=size of population
k=proportionality constant

Model:

$$\frac{dP}{dt} \propto P \qquad \text{or} \qquad \frac{dP}{dt} = kP \qquad \text{for } k > 0$$

Predictions of the model:

Example 2: Single Population, Limited Growth

Assumptions:

- If the population is small, the population grows at a rate proportional to the size of the population.
- If the population is too large, the population will decrease.

Quantities:

t=time

P=size of population

k=growth rate coefficient for small population

N=maximum size of population before growth negative

Model:

$$\frac{dP}{dt} = kP \times (\text{something})$$

where 'something' is ≈ 1 if P is small and 'something' is negative if $P > N$.

For example:

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right), \qquad k > 0$$

Predictions of the model:

Example 3: Two Populations (Predator-Prey)

Quantities:

t=time

R=size of rabbit population

H=size of hawk population

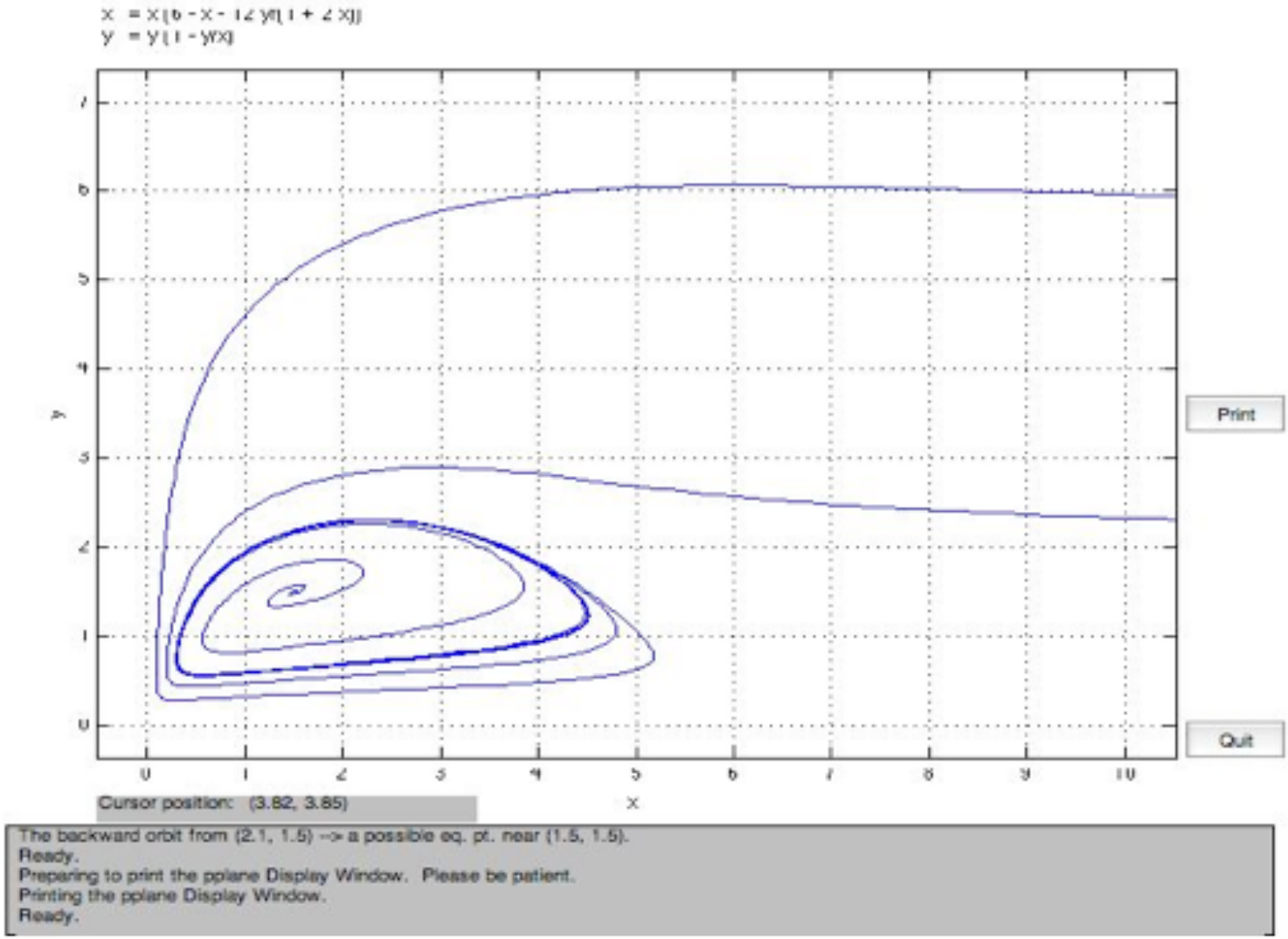
a, b, c, f are constants

Model:

$$\frac{dR}{dt} = aR\left(1 - R - \frac{bH}{1+2R}\right)$$

$$\frac{dH}{dt} = cH\left(1 - \frac{fH}{R}\right)$$

Output from numerical integration of model using the Matlab program pplane:



Important ideas from today:

- Many phenomena in the real world can be modelled with differential equations. Maths 260 looks at methods for finding out about solutions to these equations.

- We will use analytic, qualitative and numerical methods for getting information about solutions to differential equations. The main skill you will learn is how to pick and use appropriate methods for each differential equation.