DEPARTMENT OF MATHEMATICS

MATHS 190 Tutorial 2

Tutorials in Maths 190 are **collaborative tutorials**. You should work in groups of 3 or 4 students, discussing the situations and puzzles listed below, or issues arising from lectures. Part of your final mark depends on your participation in tutorials. A written solution for question 5 should be handed in for marking with Assignment 1 (due 17th March).

1. Express the following numbers as a product of prime numbers: 6, 24, 27, 35, 120.

2. A sequence of numbers uses the Fibonacci rule: $S_{n+2} = S_{n+1} + S_n$. Two terms in the sequence are

$$\ldots, 17, 28, \ldots$$

Find all positive terms **before** these numbers in the sequence.

3. **Write up your answer to this question and hand it in with your answers to Assignment 1 (due 17th March).** See below for instructions on writing up tutorial questions. Don't forget to write down the names of the people in your tutorial group, so that you can acknowledge your collaborators in your report.

Consider the rabbits problem we discussed in lecture 3: suppose we have a pair of baby rabbits, one male, and one female. Assume that rabbits can start to reproduce when they are one month old, and they have a one month gestation period. Once they start reproducing, they produce a pair of bunnies each month (one of each sex). Assume also that no pair ever dies.

(a) Make a chart as follows to show how many bunnies there are in a given month, and fill it out for at least 10 months. Write down a recurrence relation for the number of pairs of rabbits in a given month.

Time in months	Pairs of baby rabbits	Pairs of mature rabbits	Total number of pairs of rabbits
0	1	0	1
1	0	1	1
2	1	1	
3			

(b) What happens if it takes the rabbits two months before they are mature, and start to reproduce? Make a new chart for this situation, again for 10 months. *Hint: you may find it useful to include an extra column in your table.*

(c) Try to spot a pattern and write down a recurrence relation to describe the number of pairs of rabbits in a given month in this new situation.

4. Prove by contradiction (see lecture 4) that for an integer n, if n^2 is even, then n must be even. Write a very careful description of your proof. *Hint: begin by assuming the opposite: that n must be odd, then write $n = 2m + 1$ for some other integer m.*

5. If p is a prime number greater than 2, can $p + 1$ be prime? Why did I need the condition that $p > 2$?

6. Twin primes are pairs of primes of the form $\{p, p+2\}$. We could define **triplet** primes as sets of primes of the form $\{p, p + 2, p + 4\}$.

   (a) Pick any number n and show that one of the three numbers $\{n, n+2, n+4\}$ is a multiple of 3. Repeat for five other values of n.

   (b) Show that (a) is true for *any* integer value of n.

   (c) What does this mean about the number of triplet primes?

   (d) Are there any "quadruplet" primes? Justify your answer.

7. **Harder: only attempt if you have finished all the other puzzles**. Express the first 15 even numbers greater than 2 as the sum of two prime numbers. Is every even number (greater than 2) the sum of two primes?

Writing up tutorial questions

A written solution for question 3 should be handed in for marking with Assignment 1 (due 17th March). Instructions on how to hand in your assignment are on the Assignment 1 question sheet.

In your solution, you should include:

- the names of the people you discussed this with in your tutorial group;
- a clear statement of your solution to the puzzle;
- a clear explanation (in one or two paragraphs) or how you arrived at this solution; and,
- a statement of any assumptions you had to make in obtaining your answer.